

Worksheet 6

Objectives: Do pivoted LU with pencil and paper, compare cost of LA algorithms, apply Sherman-Morrison.

Problem 1: Linear algebra cost

(a) LU uses $O(n^3)$ multiplications. Considering the algorithm, why is 3 the right power of n ?

(b) To compute A^{-1} , you can use LU and forward/backward substitution to solve the matrix equation $AX = I$ column by column.

(I.e. first solve $Ax_1 = (1, 0, \dots, 0)^T$, then $Ax_2 = (0, 1, 0, \dots, 0)^T$, ..., and then assemble $A^{-1} = X = (x_1, x_2, \dots, x^n)$.)

How many multiplications (to leading order) does this process use?

(c) Suppose you had computed A^{-1} as an $n \times n$ matrix.

What is the number of multiplications used to compute $A^{-1}b$?

Problem 2: More LA cost

(a) Which of the following is cheaper (by number of multiplications)?

- Compute, apply $A^{-1}b$.
- Compute $A = LU$, use fw/bw substitution on b .

(b) Which of the alternatives from (a) is cheaper if you want to solve n linear systems with the same matrix?