Numerical Analysis (CS 450)

Worksheet 7

Objectives: **(1)** Understand costs of linear algebra operations, **(2)** set up least squares problems, **(3)** understand geometric intuition behind least squares, **(3)** be familiar with the normal equations.

Problem 1: Set up linear least squares

For each day since the semester started, you've written down

- how grumpy you were on day i of the semester, as a value g_i on a scale of 1 to 10,
- the average temperature outside on day number i, t_i, in degrees Fahrenheit.

(a) You conjecture that a linear relationship exists: $g_i \approx \alpha t_i + \beta$.

Set up a least squares problem to find an estimate of α and β, written in matrix form $Ax \cong b$. What are A, x, and b?

(b) What would A, x, and b be for a quadratic relationship $g_i \approx \alpha t_i^2 + \beta t_i + \gamma$?

(c) You come to the conclusion that neither linear nor quadratic relationships are strong enough, and that a higher (but unknown) power of t_i is needed to describe your grumpiness: $g_i \approx \alpha t_i^{\beta}$.

Take the log of the equations and write the problem in matrix form $Ax \cong b$. What are A, x, and b?

Problem 2: Derive the normal equations

Consider $\varphi(x) = \|r(x)\|_2^2 = \|Ax - b\|^2$. Expand φ and take the gradient in x to derive the normal equations.