Numerical Analysis (CS 450)

Worksheet 15

Objectives: (1) Use Arnoldi iteration to construct a similarity transform to upper Hessenberg form **(2)** Know what Ritz values are **(3)** Understand the contraction mapping theorem

Problem 1: Krylov space methods

(a) Write down a basis of k vectors that spans the Krylov space of a matrix A with respect to an initial vector x_0.

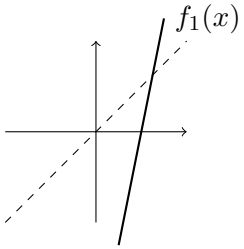
(b) What is the matrix representing the linear mapping $x \mapsto Ax$ with respect to the basis of the Krylov space from part (a)?

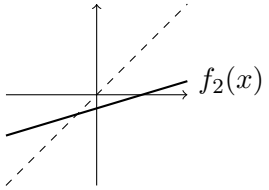
Problem 2: Root finding

(a) What is the statement of the contraction mapping theorem for the example of a map, as discussed in class?

(b) What does it mean for the value of $f(x^*)$ if x^* is a fixed point of the function $g(x) = x - f(x)$?

(c) Draw the inverse function for each of the functions in the plots shown below. Is evaluating each function well-conditioned? Is finding a root of each function well-conditioned?





(d) Decide if the marked roots of the plotted functions have multiplicity $m > 1$ or are simple roots.