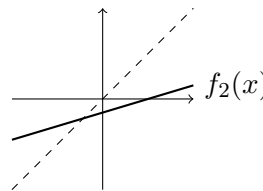
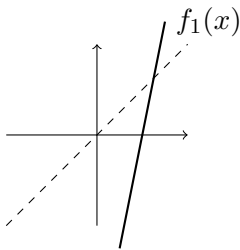


Worksheet 16

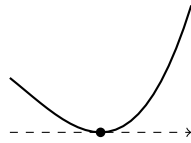
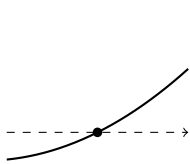
Objectives: (1) Understand the contraction mapping theorem (2) Understand sensitivity of root finding (3) Understand convergence behavior of fixed point iteration

Problem 1: Root finding

- (a) What is the statement of the contraction mapping theorem for the example of a map, as discussed in class?
- (b) What does it mean for the value of $f(x^*)$ if x^* is a fixed point of the function $g(x) = x - f(x)$?
- (c) Draw the inverse function for each of the functions in the plots shown below. Is evaluating each function well-conditioned? Is finding a root of each function well-conditioned?



- (d) Decide if the marked roots of the plotted functions have multiplicity $m > 1$ or are simple roots.



Problem 2: Convergence

- (a) Identify the approximate convergence rates of each of these iteration procedures:

1.75056	1.42	0.331
0.26258	0.304	0.166
0.03938	0.0138	0.0828
0.00590	2.87e-05	0.0414
0.00088	1.24e-10	0.0207

- (b) Suppose a fixed point iteration with iteration function g converges linearly. What determines the constant C in $\|e_{k+1}\| \approx C\|e_k\|$?