Numerical Analysis (CS 450)

Worksheet 17

Objectives: (1) When does fixed point iteration converge? **(2)** When does it converge linearly? When quadratically? **(3)** When does Newton's method converge, and how quickly?

Problem 1: Fixed point iteration

(a) Suppose $\|e_7\| = 2 \cdot 10^{-2}$ and $\|e_8\| = 10^{-2}$.

How large (approximately) would you expect $\|e_9\|$ to be if convergence is (now) asymptotic and quadratic?

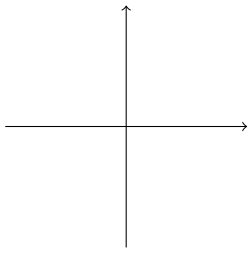
(b) How large would you expect $\|e_9\|$ to be if convergence is asymptotic and linear?

(c) Suppose a fixed point iteration with iteration function g converges linearly. What determines the constant C in $\|e_{k+1}\| \approx C\|e_k\|$?

Problem 2: Newton's method

(a) Think of a function where Newton's method will not converge. Draw a sketch below. Also mark your initial guess for Newton.

Hint: $f'(x_k)$ is in the denominator. What type of number is bad in a denominator?



(b) Let x_k and x_{k-1} be the current and previous iterates in the Newton and secant methods.

Write down the estimated slope near x_k used in each method.