Objectives: (1) Understand, predict convergence behavior of Newton in 1D/nD (2) Understand optimization terminology (3) Understand existence/uniqueness/sensitivity results for optimization problems

Problem 1: Newton's method

- (a) Think of a function where Newton's method will not converge. Draw a sketch below. Also mark your initial guess for Newton.
 - *Hint:* $f'(x_k)$ is in the denominator. What type of number is bad in a denominator?



(b) Let x_k and x_{k-1} be the current and previous iterates in the Newton and secant methods. Write down the estimated slope near x_k used in each method.

Problem 2: Existence and Uniqueness of minimizers

(a) Are the following functions coercive? convex? strictly convex?



- (b) What does the coercivity, convexity, and strict convexity of the functions mean for the minimizers?
- (c) Suppose $\nabla f(x^*) = 0$ and $H_f(x^*)$ is negative definite. (i.e. $-H_f(x^*)$ is positive definite) What does f look like near x^* ?