

Worksheet 23

Objectives: (1) Understand how to deal with equality and inequality constraints (2) Use linear systems to do interpolation (3) Be able to use the Lagrange basis

Problem 1: Constrained optimization

(a) What is the Lagrangian function for the following problem?

$$\begin{aligned} & \min_{(x,y)} x^2 + y^2 + z^2 \\ & \text{subject to } x + y - 1 = 0 \\ & \text{and } x + z \leq 0 \end{aligned}$$

(b) What system of equations would you consider to solve this constrained optimization problem?

(c) Are the solutions of this system guaranteed to be local minima of the constrained optimization problem?

Problem 2: Interpolation

(a) Which of the following is *not* a good application of interpolation?

- (a) Smoothing the error in noisy data
- (b) Approximating a complicated function by a simple one
- (c) Computing unknown values in between known values on a table
- (d) Replacing a collection of data points by a smooth curve

(b) Suppose you have the function

$$f(x) = 2 - 3x + 4x^2$$

Write this function as a linear combination of three basis functions (you pick). What do your basis functions *span*?

(c) What is the degree of the j -th Lagrange basis function for interpolating n data points?