Numerical Analysis (CS 450)

Worksheet 26

Objectives: (1) Understand conditioning of the quadrature problem **(2)** Be able to use the method of undetermined coefficients to derive Newton-Cotes quadrature rules **(3)** Understand accuracy and stability results for (Newton-Cotes) quadrature

Problem 1: Newton-Cotes Quadrature

(a) Write the linear system for the method of undetermined coefficients in $\boldsymbol{Aw} = \boldsymbol{b}$ form. For $n = 4$, write down the expressions for $\boldsymbol{A}$ and $\boldsymbol{b}$. Does the matrix look familiar?

(b) For the following quadrature rules, if the length of the integration interval decreases by a factor of 2, by what factor does the relative error in the quadrature result decrease?

Midpoint rule:

Trapezoidal rule:

Simpson's rule:

Problem 2: Gaussian Quadrature

Suppose you have a Gaussian quadrature rule on the interval $(-1, 1)$ with weights w_i and nodes x_i for $i = 1, \ldots, n$, but you would like to integrate a function f on the interval (a, b) instead.

Write down an expression for a Gaussian quadrature rule on (a, b):