Numerical Analysis (CS 450)

Worksheet 27

Objectives: **(1)** Understand accuracy of composite quadrature **(2)** Be able to find weights of, know properties of and use a Gaussian quadrature **(3)** Understand drawbacks of numerical differentiation **(4)** Be able to apply finite differences (if absolutely needed)

Problem 1: Gaussian Quadrature

(a) Suppose you have a Gaussian quadrature rule on the interval $(-1, 1)$ with weights w_i and nodes x_i for $i = 1, \ldots, n$, but you would like to integrate a function f on the interval (a, b) instead.

Write down an expression for a Gaussian quadrature rule on (a, b):

(b) What is the largest polynomial degree that is integrated exactly by a two-point Gaussian quadrature rule?

(c) Suppose you needed to integrate data at *given* integration nodes. You do the following, on a number of subintervals:

1. Interpolate using cubic polynomials from the given points to Gauss points.
2. Use 4-point Gaussian quadrature for accurate evaluation of the integrals.

Does this procedure make sense? Why?/why not?

(d) If you needed to compute

$$\int_{-1}^{1}\int_{-1}^{1} \sin(x \cdot y)dxdy$$

how might you use Gaussian quadrature to do so?