Numerical Analysis (CS 450)

Worksheet 28

Objectives: (1) Understand drawbacks of numerical differentiation **(2)** Be able to apply finite differences (if absolutely needed) **(3)** Be able to apply Richardson extrapolation **(4)** Know the following terms as applied to ODEs: IVP, order, autonomous, homogeneous

Problem 1: Numerical differentiation, Richardson

(a) Name three reasons why numerical differentiation can backfire.

(b) Suppose you have two (first-order) finite difference approximations:

$$f'(x) = \underbrace{\frac{f(x+2h)-f(x)}{2h}}_{A} + a \cdot 2h + O(h^2), \qquad f'(x) = \underbrace{\frac{f(x+h)-f(x)}{h}}_{B} + ah + O(h^2).$$

Find factors α and β with $\alpha + \beta = 1$ such that the term with a cancels in $\alpha A + \beta B$.

(c) Suppose your α, β do not obey $\alpha+\beta = 1$. For example, use $\alpha = 1 = \beta$. Is still $f'(x) \approx \alpha A + \beta B$?

Problem 2: ODEs

(a) Rewrite the following ODE as an autonomous first-order system:

$$y''(t) = 5t^{2y(t)}.$$

(b) Which of the following systems of ODEs is stable?

$$y'(t) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} y(t) \qquad y'(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} y(t) \qquad y'(t) = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} y(t) \qquad y'(t) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} y(t)$$