Numerical Analysis (CS 450) Worksheet 29

Objectives: (1) Know the following terms as applied to ODEs: IVP, order, autonomous, homogeneous (2) Know how to apply stability regions

Problem 1: ODEs

(a) Rewrite the following ODE as an autonomous first-order system:

$$y''(t) = 5t^{2y(t)}.$$

(b) Which of the following systems of ODEs is stable?

$$y'(t) = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} y(t) \qquad y'(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} y(t) \qquad y'(t) = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix} y(t) \qquad y'(t) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} y(t)$$

Problem 2: Euler method

(a) What is the solution to

and

and

$$u' = -100u$$

u' = -u

u' = -10u

with u(0) = 1? Sketch them.

(b) What size time step does the Euler method need for each of the previous problems?