Numerical Analysis (CS 450)

Worksheet 31

Objectives: (1) Read and write Butcher tableaux **(2)** Make use of stability region plots **(3)** Understand terminology, existence, and uniqueness of BVPs.

Problem 1: Butcher tableaux, stability regions

(a) Write a Butcher tableau for forward and backward Euler.

(b) Write a Butcher tableau for the *midpoint method*:

$$y_{k+1} = y_k + hf\left(t_k + \frac{h}{2}, y_k + \frac{h}{2} f(t_k, y_k)\right)$$

(c) Write a Butcher tableau for an implicit method using the trapezoidal rule.

(d) Consider a linear, constant-coefficient system of ODEs $\boldsymbol{y}' = \boldsymbol{A}\boldsymbol{y}$ in which $\boldsymbol{A}$ has an eigenvalue $-3 + 4i$. Give an approximate step size h so that forward Euler will be stable for this system (only considering this eigenvalue).