Integral Equations and Fast Algorithms
Lecture 23: Nyström/projection error analysis

CS 598AK · November 12, 2013

Outline

IE discretization: Nyström

IE discretization: Projection

Compactness-based convergence

X Banach space (think: of functions)

Theorem (Not-quite-norm convergence [Kress LIE Cor 10.4])

$A_n : X \to X$ bounded linear operators,
functionwise convergent to $A : X \to X$
Then convergence is uniform on compact subsets $U \subset X$, i.e.

$$\sup_{\varphi \in U} \|A_n\varphi - A\varphi\| \to 0 \qquad (n \to \infty)$$

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Only on compact subsets of X!

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Set $\mathcal{A}$ of operators $A : X \to X$

Definition (Collectively compact)

$\mathcal{A}$ is called *collectively compact* iff
for $U \subset X$ bounded, $\mathcal{A}(U)$ is relatively compact.

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How can we use the two together?

Making use of collective compactness

X Banach space, $A_n : X \to X$, (A_n) collectively compact, $A_n \to A$ functionwise.

Just showed

Corollary (Post-compact convergence [Kress LIE Cor 10.8])

- $\|(A_n - A)A\| \to 0$
- $\|(A_n - A)A_n\| \to 0$

$(n \to \infty)$

Making use of collective compactness

Assume $(I - A)^{-1}$ exists, with A compact, (A_n) collectively compact and $A_n \to A$ functionwise.

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Estimate $\|(I - A_n)^{-1}\|$.

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Let $\varphi - A\varphi = f$ and $\varphi_n - A_n\varphi_n = f_n$.
$(I - A_n)(\varphi_n - \varphi) = ?$
Derive error estimate.

Anselone's theorem

Assume $(I - A)^{-1}$ exists, with $A : X \to X$ compact, $(A_n) : X \to X$ collectively compact and $A_n \to A$ functionwise.

Theorem (Nyström error estimate [Kress LIE Thm 10.9])

For sufficiently large n, $(I - A_n)$ is invertible and

$$\|\varphi_n - \varphi\| \le \frac{1 + \|(I-A)^{-1}A_n\|}{1 - \|(I-A)^{-1}(A_n - A)A_n\|}(\|(A_n - A)\varphi\| + \|f_n - f\|)$$

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What's the key message of this estimate?

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Fun fact: Can interchange roles of A and A_n. What does that mean?

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Nyström: *specific to I + compact.* Why?

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We've *assumed* collective compactness. Do we have that?

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Assume

$$\sum |\text{quad. weights for } n \text{ points}| \leq C \qquad \text{(independent of } n\text{)} \quad (1)$$

Then $(A_n\varphi)$ bounded and equicontinuous. (Why?)

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Also assumed functionwise uniform convergence, i.e.
$\|A_n\varphi - A\varphi\| \to 0$ for each φ.
Follows from equicontinuity of $(A_n\varphi)$.

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Assumption (1) is important to make all this work!

Outline

IE discretization: Nyström

IE discretization: Projection

Céa's Lemma: Convergence for Projection

X, Y Banach spaces, $A : X \to Y$ injective, $P_n : Y_n \to Y_n$

Theorem (Céa's Lemma [Kress LIE Thm 13.6])

Convergence of the projection method
$\Leftrightarrow$ *There exist n_0 and M such that for $n \geq n_0$*

1. $P_n A : X_n \to Y_n$ *are invertible,*
2. $\|(P_n A)^{-1} P_n A\| \leq M.$

In this case,

$$\|\varphi_n - \varphi\| \leq (1 + M) \inf_{\psi \in X_n} \|\varphi - \psi\|$$

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What if we implement a perturbation?

Questions?

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