

$Ax=b$   
 $S$  set of vectors  
 $A \in$  matrix

$$AS = \{As; s \in S\}$$

$$AN(A) = \{0\}$$

$x \in N(AM)$   $\leftarrow$   $A, M$  matrices,  $M$  invertible

$$\underbrace{AM}_y x = 0 \quad \begin{matrix} \downarrow \text{via } M \\ Mx = y \end{matrix} \quad \begin{matrix} \downarrow \text{via } M \\ y \in N(A) \end{matrix} \quad \begin{matrix} \downarrow \text{via } M \\ y \neq 0 \end{matrix}$$

$MN(AM) \subseteq N(A)$

$x \in N(MA)$   $\leftarrow$   $A, M$  matrices,  $M$  invertible

$$\underbrace{MA}_y x = 0 \quad \begin{matrix} y=0 \\ \Rightarrow Ax=0 \\ \Rightarrow x \in N(A) \end{matrix}$$

$$N(MA) \subseteq N(A)$$

$\swarrow$  permutation matrices

$$ABP x = Dy \quad | \quad A^{-1}$$

$$\Leftrightarrow \cancel{A} ABP x = A^{-1} Dy \quad | \quad B^{-1}$$

$ab = ba$   
 $5 \cdot 7 = 7 \cdot 5$   
 $AB + BA$

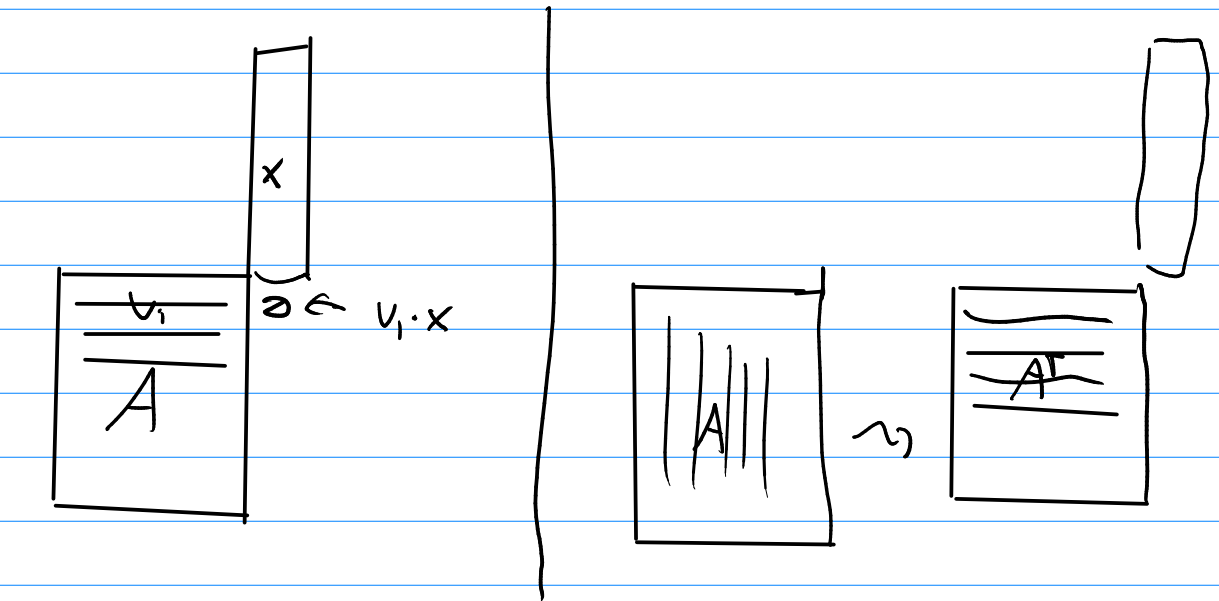
$$P_x = B^{-1} A^{-1} D y \quad | \quad P^{-1} \equiv P^T$$

$$x = P^T B^{-1} A^{-1} D y$$

$$ABP = 0 \quad |^T$$

$$\Leftrightarrow (ABP)^T = 0^T$$

$$\Leftrightarrow P^T B^T A^T = 0^T$$



$$PA^T = LU \quad | \quad P^{-1} \quad \left| \quad PA^T = LU \quad | \cdot P^{-1} \right.$$

$$\Leftrightarrow \cancel{P^T} P A^T = P^T L U \quad | \quad P^T \quad \left| \quad P A^T P^T = L U P^{-1} \right.$$

$$\Leftrightarrow A = (P^T L U)^T$$

$$= U^T L^T P$$

$$\Rightarrow N(A) = N(U^T L^T P)$$

$$U = \begin{pmatrix} \diagup & \diagup & \diagup & \diagup & \diagup \\ & \diagup & \diagup & \diagup & \diagup \\ & & \diagup & \diagup & \diagup \\ & & & \diagup & \diagup \\ & & & & 0 \end{pmatrix} \quad U^T = \begin{pmatrix} \diagdown & \diagdown & \diagdown & \diagdown & \diagdown \\ & \diagdown & \diagdown & \diagdown & \diagdown \\ & & \diagdown & \diagdown & \diagdown \\ & & & \diagdown & \diagdown \\ & & & & 0 \end{pmatrix}$$

$$x \in N(U^T) \Leftrightarrow U^T x = 0$$

$$y \in \underbrace{N(U^T L^T P)}_{N(A)} \Leftrightarrow \underbrace{U^T L^T P}_{x} y = 0$$

$$A: -x - v + 800 = 0$$

$$A: \rightarrow \begin{pmatrix} x & y & z & v \\ -1 & & & -1 \end{pmatrix} = \begin{pmatrix} -800 \end{pmatrix}$$

$$L^T P y = x \quad | \quad L^T$$

$$P y = L^{-T} x \quad | \quad P^T$$

$$y = P^T L^{-T} x$$

$$N(A) = P^T L^{-T} N(U^T)$$

## ⑥ Orthogonality

Definition: An inner product  $(\cdot, \cdot)$  is a function from  $V \times V \rightarrow$  scalars that obeys

$$(\alpha x, y) = \alpha(x, y)$$

$$(x+y, z) = (x, z) + (y, z)$$

$$(x, y) = (y, x) \quad \rightsquigarrow \quad (x, \alpha y) = \alpha(x, y)$$

$$(x, x) \geq 0 \quad (x, x) = 0 \Leftrightarrow x = 0$$