einsum("$\overbrace{ijk}^{\substack{\text{(1 per axis of a)} \\ \text{3 letters}}}$, $\overbrace{jlin}^{\text{4 letters}}$ → $\overbrace{jln}^{\substack{\text{(1 per axis of b)} \\ \text{output}}}$",

a, b)

10×20×13 20×5×10×7

$$c_{jln} = \sum_{ik} a_{ijk} \, b_{jlin}$$

einsum("ijk, jlin →", a, b)

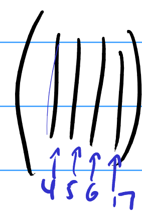
$$c = \sum_{ijkln} a_{ijk} \, b_{jlin}$$

einsum("i→", a) ← sums entries of vec

$$c = \sum_i a_i$$

a[$\overset{xyz}{\text{ind. into vector}}$, $\overset{0..3}{\text{ind of vector}}$]

v[ind of vector]

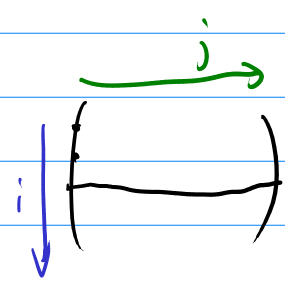


$$c_i = \sum_j a[i,j] \cdot v[j]$$

einsum("ij, j → i", a, v)

a[i,j]

2

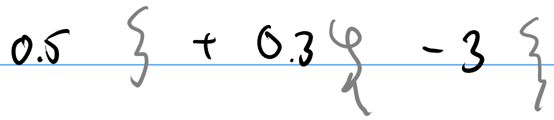


$$\text{Span}\{v_1, v_2, v_3, \ldots, v_n\} = \{\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \cdots + \alpha_n v_n : \alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_n \in \mathbb{R}\}$$

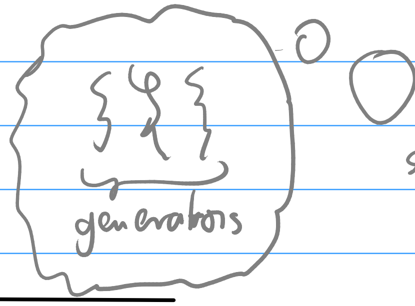
a vector space

or other numbers

$1 \cdot v = v$ $\qquad$ $v + (-1) \cdot v = 0$



$$\underbrace{\begin{pmatrix}0.5\\0.3\\-3\end{pmatrix}}_{\text{coordinate}}$$



swept under the rug.

$$0.5\begin{pmatrix}3\\4\\7\end{pmatrix}+0.3\begin{pmatrix}2\\1\\4\end{pmatrix}-3\begin{pmatrix}5\\7\\1\end{pmatrix}$$

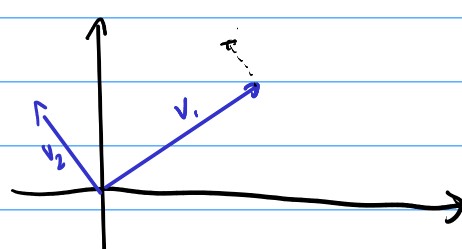
$$\underbrace{\begin{pmatrix}0.5\\0.3\\-3\end{pmatrix}}_{\text{coord. "quantities"}} \qquad \underbrace{\begin{pmatrix}3\\4\\7\end{pmatrix}\begin{pmatrix}2\\1\\4\end{pmatrix}\begin{pmatrix}5\\7\\1\end{pmatrix}}_{\text{gen. "ingredients"}}$$

$$\cancel{\begin{pmatrix}3\\4\\7\end{pmatrix}+\begin{pmatrix}0.5\\0.3\\-3\end{pmatrix}}$$

⚠ If the generators are $\begin{pmatrix}1\\0\\0\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}$, then coordinate vectors and linear comb. given by them coincide.

$$0.5\begin{pmatrix}1\\0\\0\end{pmatrix}+0.3\begin{pmatrix}0\\1\\0\end{pmatrix}-3\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}0.5\\0.3\\-3\end{pmatrix}$$

The only case where coord. vec. and actual vectors coincide.



$\begin{pmatrix}1\\0.5\end{pmatrix}$

Example:

with respect to

Given a coord. vec. $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ wrt the generators

$$g_1 = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \quad g_2 = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

(a) what's the underlying vector?

$$v = 2 \cdot \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} + 6 \cdot \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

(b) What's the coord. vector of v wrt. the standard generators

$$2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ ← coord. vec.

② Linear functions

$$f: V \to W$$

- $f(x+y) = f(x) + f(y)$
- $f(\alpha x) = \alpha f(x)$

$$f \begin{cases} V: & v_1, \dots, v_m \\ W: & w_1, \dots, w_n \end{cases}$$

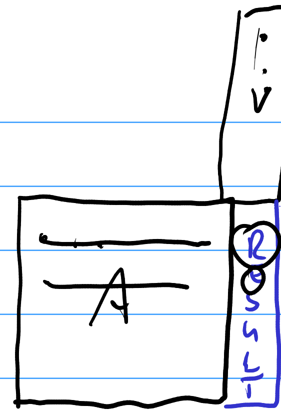
$$x = \alpha_1 v_1 + \cdots + \alpha_m v_m$$

$$f(x) = \alpha_1 \underbrace{f(v_1)}_{\beta_{11} w_1 + \cdots + \beta_{n1} w_n \in W} + \cdots + \alpha_m \underbrace{f(v_m)}_{\beta_{1m} w_1 + \cdots + \beta_{nm} w_n \in W}$$

$$\begin{pmatrix} \beta_{11} & \cdots & \beta_{1m} \\ \vdots & \ddots & \vdots \\ \beta_{n1} & \cdots & \beta_{nm} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{pmatrix} = \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_n \end{pmatrix}$$

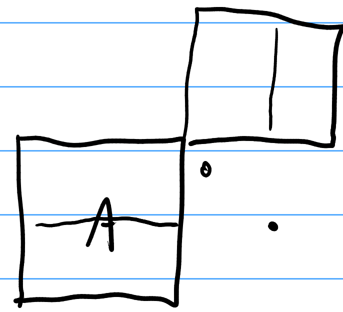
$$f(x) = \underbrace{\gamma_1}_{\beta_{11} \cdot \alpha_1 + \beta_{12} \cdot \alpha_2 + \cdots + \beta_{1m} \alpha_m} w_1 + \gamma_2 w_2 + \cdots + \gamma_n w_n$$

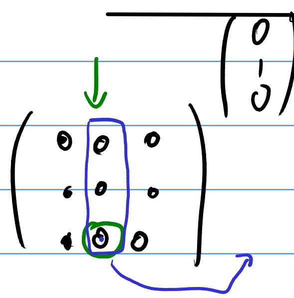
$A \cdot v$



$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

AB

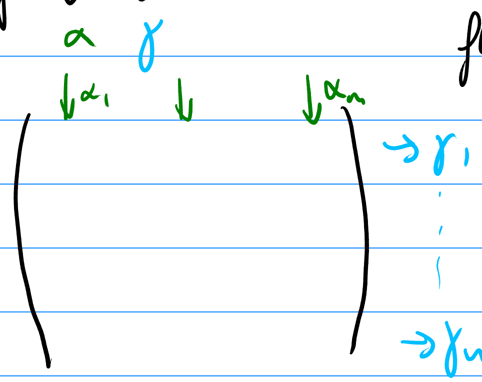




$f : V \to W$

$x = \alpha_1 v_1 + \dots + \alpha_m v_m$

$f(x) = \gamma_1 v_1 + \dots + \gamma_n w_n$



$f(v_1) = v_2$