$f : V \to W$

$f(1 \cdot v_1) = 2w_3$

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & & ? \\ 0 & & \\ 2 & & \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$$

$f : V \to V$

v_1, v_2, v_3

$(\alpha_1, \alpha_2, \alpha_3)$

$\hookrightarrow \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_3 \end{pmatrix} \rightsquigarrow$ coord wrt. w_1, w_2, w_3

$$\begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 7 & 1 \\ & 4 & 2 \\ & & 15 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \rightsquigarrow$$

$$\begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 7 & 1 \\ & 4 & 2 \\ & & 15 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 15 \end{pmatrix}$$

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 7 & 1 \\ 0 & 4 & 2 \\ & & 15 \end{pmatrix} \begin{pmatrix} \alpha_2 \cdot 4 + \alpha_3 \cdot 2 \end{pmatrix}$$

Chaining linear functions

$f: V \to W \qquad g: W \to X \qquad$ both linear

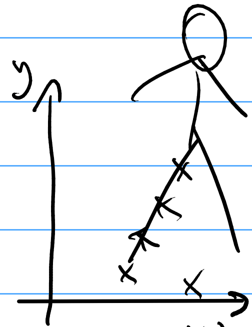
$$f(x) = Ax \qquad g(x) = Bx$$

$g(f(x))$ repr. by $\underbrace{B(Ax)}$.

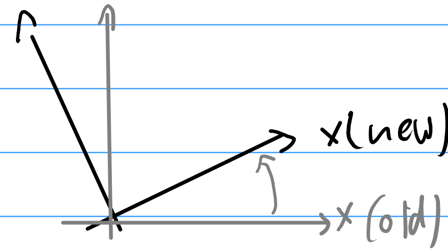
$$\|$$

"associativity" $\qquad \underline{(BA)x} = BAx$ because of assoc.

$$(BA)_{ij} = \sum_k A_{ik} B_{kj}$$



$\begin{pmatrix} x \\ y \end{pmatrix}$



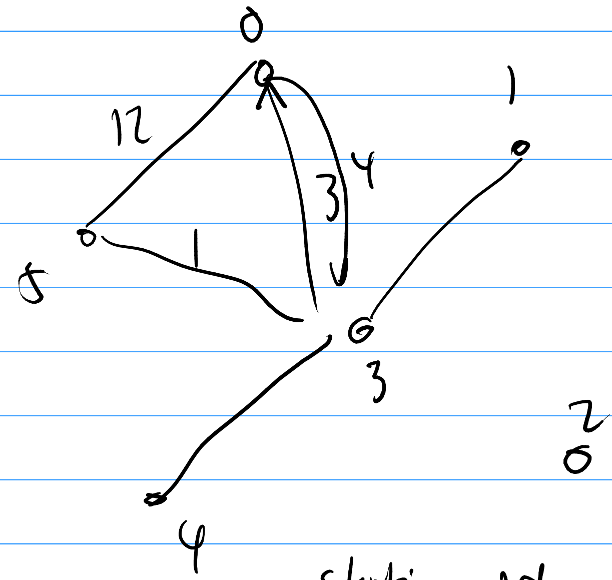
$\begin{pmatrix} (x_{new})_x & (y_{new})_x \\ (x_{new})_y & (y_{new})_y \end{pmatrix}$

$(\alpha_1\ \alpha_2\ \alpha_3)$

$\begin{pmatrix} \beta_{11} & & \\ & \ddots & \\ & & \beta_{43} \end{pmatrix} \rightarrow \gamma_1 = \alpha_1\beta_{11} + \alpha_2\beta_{12} + \alpha_3\beta_{13}$

$\gamma_i = \sum_j \beta_{ij}\alpha_j$

gamma = einsum("ij,j→i")



starting nodes

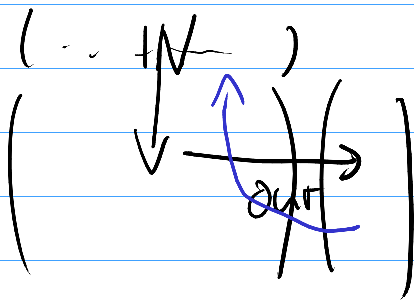
ending nodes

$$\begin{array}{c c} & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \\ & & & 4 & & \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} & & & & & \\ & & & & & \\ & & & & & \\ 3 & & & & & \\ & & & & & \\ & & & & & \end{pmatrix} \end{array}$$

$(5, 5)$

$\mathfrak{S}(5)$

$A d = \lambda d$



$$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 & 4 \\ & 1 & 7 \\ & & 5 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \begin{matrix} = \\ = 1 \cdot \alpha_2 + 7 \cdot \alpha_3 \\ = 5 \cdot \alpha_3 \end{matrix}$$

"Back-substitution"

$$\begin{pmatrix} \circ & & \\ \circ & \circ & \\ \circ & \circ & \circ \end{pmatrix}$$

"Forward substitution"

$$(AB)_{ij} = \sum_k A_{ik} B_{kj}$$ einsum("ik,kj->ij", A, B)

$$(Ax)_i = \sum_k A_{ik} x_k$$ einsum("ik,k->i", A, x