$$\begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \\ a & b & b & c \end{array}$$

$$0 \quad 1 \quad -1$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \cancel{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{array}{ccc} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} & \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \\ v_1 & v_2 & v_3 \end{array}$$

$$\begin{pmatrix} 7 \\ 0 \\ 0 \\ 4 \end{pmatrix} = 7v_2 + 4v_3$$

$$= 4v_1 + 3v_2$$

$$\begin{pmatrix} \alpha_1 & \cdots & \alpha_n \end{pmatrix}^T$$

$$\begin{pmatrix} | & | & | & & | \\ v_1 & v_2 & v_3 & \cdots & v_n \\ | & | & | & & | \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \vdots \\ \gamma_n \end{pmatrix}$$

$$\begin{array}{ccccc} \alpha_1 & \alpha_2 & & & \alpha_n \end{array}$$

$$\begin{pmatrix} x & x & x & x & x \\ & x & x & x & x \\ & & x & x & x \\ & & & \triangle & \boxed{x} \\ & & & & \otimes \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \\ \\ \\ \gamma_n \end{pmatrix}$$

$$= \triangle \cdot \alpha_{n-1} + \square \cdot \alpha_n$$

$$= \otimes \cdot \alpha_n$$

$$a_{ii} \cdot x_i + a_{i,i+1} x_{i+1} + \dots + a_{i,n} x_n = b_i$$

$$x_i = \frac{b_i - a_{i,i+1} x_{i+1} + \dots + a_{i,n} x_n}{a_{ii}}$$

(3) Basis and dimension

Definition: $v_1, \ldots, v_n$ are a basis of a v.s. V if and only if:

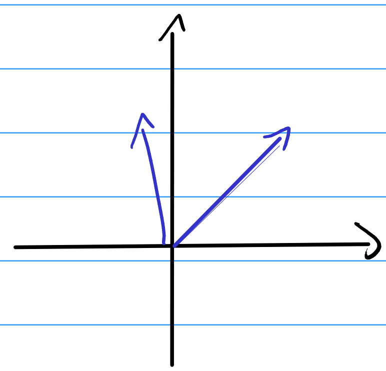
(1) linearly independent

(2) $\text{span}\{v_1, \ldots, v_n\} = V$

Facts:

- subsets of l.i. vectors are l.i.
- Can always reduce a finite set of vectors to a l.i. set.

Detour:



Drawing vectors randomly usually gives l.i. vectors (with prob. 1).

- Let V be a vector space and I have
  $S = \{v_1, v_2, \ldots, v_n\}$ with $\operatorname{span} S = V$
  $B = \{b_1, \ldots, b_m\}$ li
  then

$$\underbrace{n \geq m} \qquad \text{("Morphing lemma")}$$

- Consequence:

  All bases of a vector space have the same size.

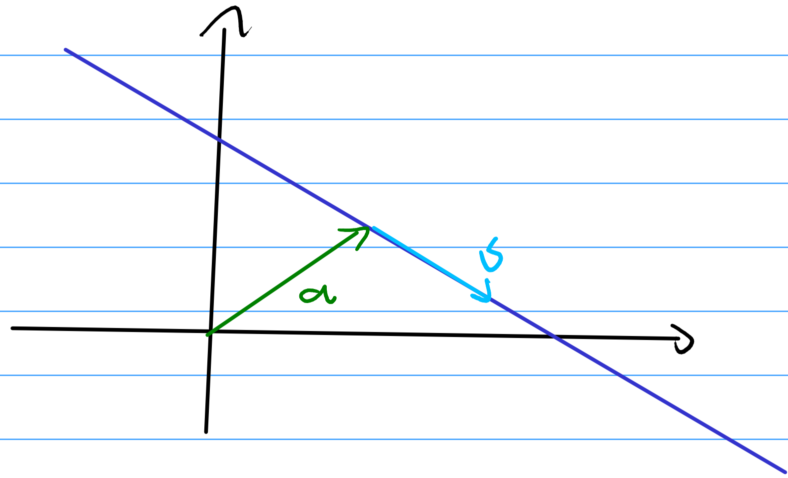
  That number is called the dimension

  $\dim V$.

- v.s. $V \subseteq W \Rightarrow \dim V \leq \dim W$

- if $V \subseteq W$ v.s and $\dim V = \dim W$, then $V = W$

$$\begin{pmatrix} 7 \\ 4 \\ 7 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 7 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 4 \\ 3 \\ 2 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$



$a + \alpha b$