$$\|(1,2,3)\| = -7$$

Norm:

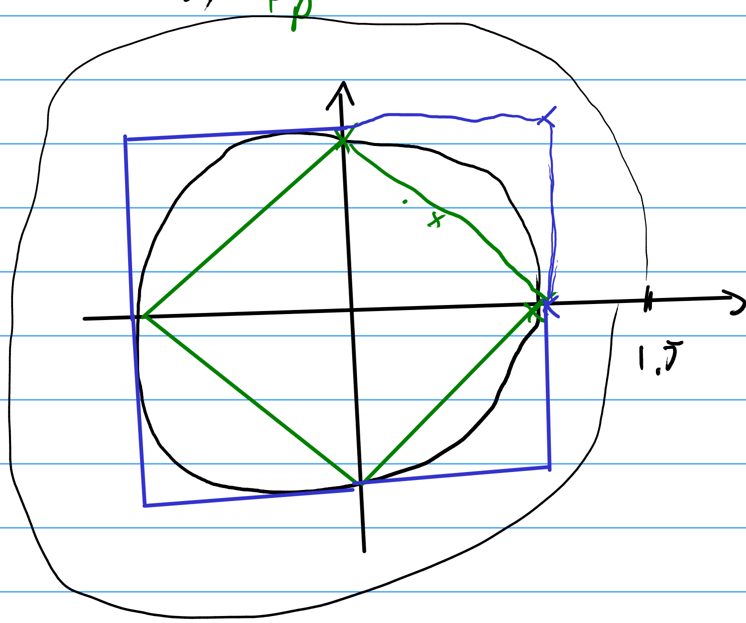
$$\|\cdot\| \geq 0$$

$$\|x\| = 0 \iff x = 0$$

$$\|\alpha x\| = |\alpha| \|x\|$$

$$\|x+y\| \leq \|x\| + \|y\|$$

$$\left\| \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \right\|_{\ell_p} = \sqrt[p]{|x_1|^p + |x_2|^p + |x_3|^p}$$



$$\|x\|_2 = \sqrt{x_1^2 + x_2^2} = 1$$

$p = 1$

$$\|x\|_1 = |x_1| + |x_2|$$

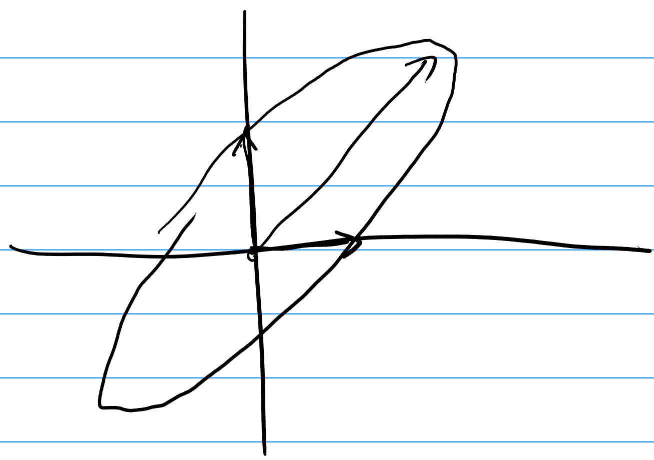
$p = \infty$

$$\|x\|_\infty = \max(|x_1|, |x_2|)$$

$$\|A\| = \max_{x \neq 0} \frac{\|Ax\|}{\|x\|} = \frac{\|f(x)\|}{\|x\|}$$

$f(v_1) = 3v_3$ Assume $\|v_i\| = 1$

$$\frac{\|3v_3\|}{\|v_1\|} = \frac{|3|\,\|v_3\|}{\|v_1\|} \approx \frac{\|(0,0,3)\|}{\|(1,0,0)\|} = \frac{3}{1}$$



5

Gaussian elimination

Row Echelon Form:

$$\begin{pmatrix} 2 & 4 & 1 & 7 & 3 & 15 \\ & 5 & 1 & 2 & 3 & 19 \\ & & & 8 & 5 & 1 \\ & & & & & 2 \end{pmatrix}$$

Gauss-Jordan elimination:

$$\left(\begin{array}{c|c} A & b \end{array}\right)$$

G-J

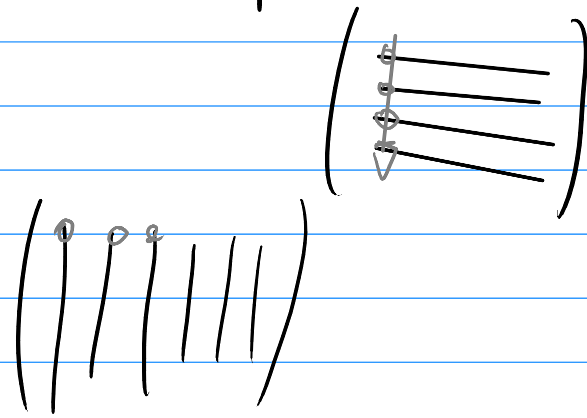
$$\left(\begin{array}{ccc|c} d_1 & & & \\ & d_2 & & \tilde{b} \\ & & d_3 & \end{array}\right)$$

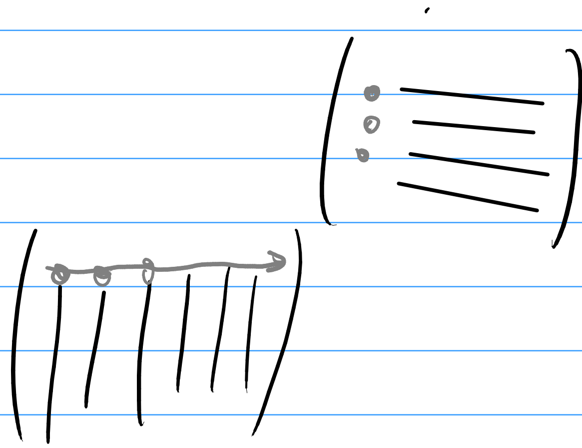
$$\left(\begin{array}{c|c} A & I \end{array}\right)$$

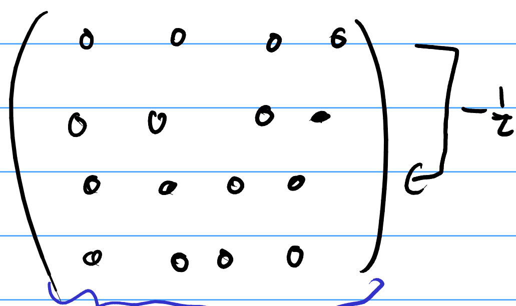
G-J.

$$\left(\begin{array}{c|c} I & A^{-1} \end{array}\right)$$

Matrix-matrix multiplication:







A

$$\underbrace{\begin{pmatrix} 1 & & & \\ & 1 & & \\ -\frac{1}{2} & & 1 & \\ & & & 1 \end{pmatrix}}_{M} A \quad \leftarrow \text{elimination matrices}$$

$$M^{-1} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ \frac{1}{2} & & 1 & \\ & & & 1 \end{pmatrix}$$

Try to build Gaussian elim w/ elim matrices

$$\underbrace{M_\ell \cdots M_4 M_3 M_2 M_1}_{\text{lower triangular matrix}} A = U$$

$$A = \underbrace{M_1^{-1} M_2^{-1} \cdots \cdots M_\ell^{-1} M_\ell^{-1}}_{\text{lower triangular}} U$$

$$A = L\,U$$

<u>LU breakable:</u>

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \to$ just swap rows

<u>Partial pivoting</u> Swap the row with the entry of largest magnitude (in the current column) to the top

$$M_6 P_3 M_5 M_4 P_2 M_3 M_2 M_1 P_1 A = U$$

$$PA = LU$$

$$P_3 P_2 P_1 A$$