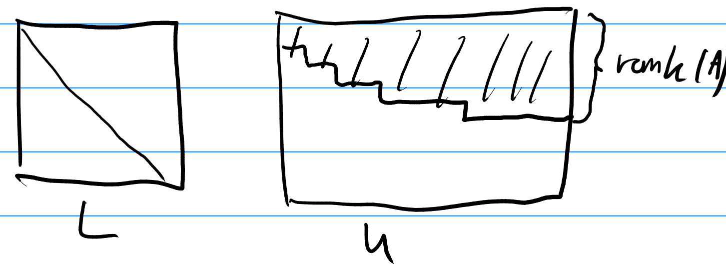
rank = dim col space = dim row space

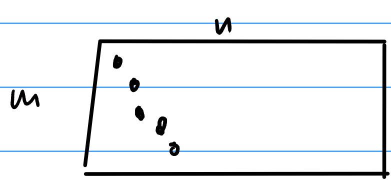
rank + dim $N(A)$ = # columns

$A =$ 

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ← no LU if unpermuted

rank + dim $N(A) = n$

m



rank? $= m$

dim $N(A) = n - m$

rank? n

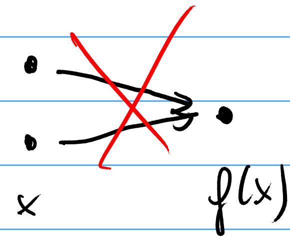
dim $N(A) = 0$

Properties of linear functions

$f: V \to W$ linear $\dim(V)=n$ $\quad \dim(W)=m$

matrix A represents f: $\quad m \times n$

one-to-one:



$f(x)=f(y) \Rightarrow x=y$

$f(\alpha x)=\alpha f(x)$

$f(0)=0$.

$f(x)=f(y) \Leftrightarrow f(x)-f(y)=0 \Leftrightarrow f(x-y)=0$

for a lin function: equivalent to

$f(x)=0 \Leftrightarrow x=0$

$\Rightarrow$!

one-to-one $\Leftrightarrow$ $\dim N(f)=0$

onto: for every $w \in W$, I can find a $v \in V$ s.t. $f(v)=w$.

$\text{rank}(f) = \dim(W)$

$\uparrow$

dim colspace(A)

invertible: $\quad$ one-to-one + onto

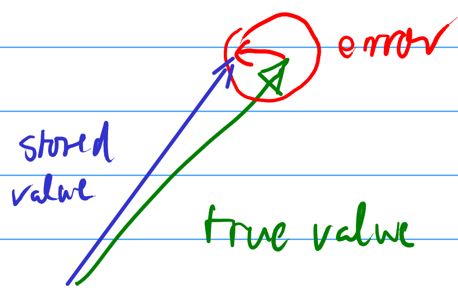
$\dim N(A)=0 \qquad \text{rank}(A)=m$

$$\underset{m}{\text{rank}} + \underset{0}{\dim N(A)} = n$$

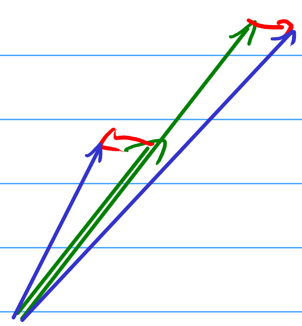
$$f^{-1} \quad , \quad A^{-1}$$

$$M A = U$$

Testing for linear independence using computers



stored value $\approx$ true value + error



Computational expense of LU

$M_\ell \cdots M_3 M_2 M_1 A$ $\qquad\qquad A: n \times n$

$\ell \sim n^2$

comp. expense of matrix · matrix:

$$(AB)_{ij} = \sum_k A_{ik} B_{kj} \quad \rightsquigarrow n^3$$

Matrix-matrix products carried out naively: n^5

With "smart" elimination matrices: n^3

Cost of LU.

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