$A = 30 \cdot Id$ $\qquad \|A\| = |30| \, \|Id\|$

$A^{-1} = \frac{1}{30} \cdot Id$ $\qquad \|A^{-1}\| = |\frac{1}{30}| \cdot \|Id\|$

---

orthogonal vectors after G-S

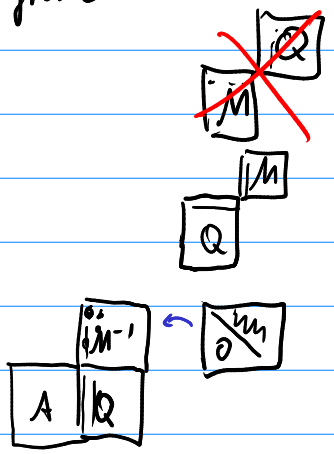
$A = \cancel{M} \, \cancel{Q}$

$= Q M$

$A M^{-1} = Q$

$A = Q \overset{R}{M}$

$Q^T A = R$



Can we solve $Ax = b$ using QR?

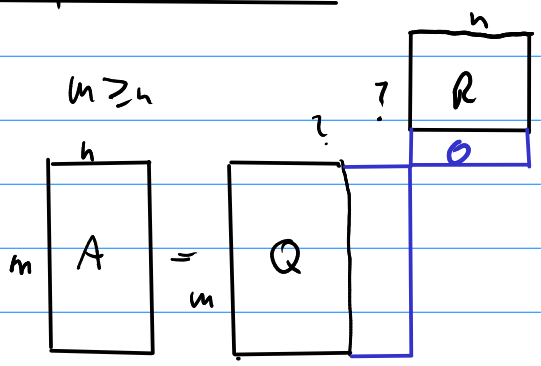
$Ax = b \rightsquigarrow QRx = b \rightsquigarrow Qy = b$

$y = Q^T b \leftarrow n^2$

$Rx = y \rightsquigarrow$ backsubst n^2

Non-square matrices

SparseLU: UMFPACK
SuperLU



Either "full" / "compl" QR

Q m × m
R m × n

or "reduced" QR

Gram-Schmidt ⟶ Q m × n
R n × n

What happens to the 2-norm of a vector if I apply an orthogonal matrix?

↳ square

$$\|Qx\|_2^2 = (Qx)\cdot(Qx) = (Qx)^T Qx$$
$$= x^T Q^T Q x$$
$$= x^T x$$
$$= \|x\|_2^2$$

(↑ orthogonal)

"Solving" overdetermined linear systems

$Ax = b + r$

A tall

Skinny

→ more eqns than unknowns
→ bad news

$$\min_x \|Ax - b\|_2$$

(↑ residual)

$$\min_x \|Ax-b\|_2^2 = (Ax-b)_1^2 + (Ax-b)_2^2 + \cdots + (Ax-b)_4^2$$

least squares

full QR

$$\|Ax-b\|_2^2 = \|QRx-b\|_2^2$$

$$= \|Q^T(QRx-b)\|_2^2$$

$$= \|Q^TQRx - Q^Tb\|_2^2$$

$$= \|Rx - Q^Tb\|^2$$