$Ax = U \Sigma V^T x$

$m \times n$

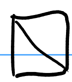
$B A B^{-1}$

$\|A\|_2 = \max \frac{\|Ax\|_2}{\|x\|_2}$

If A square and inv. $\quad V^{-T} = (V^{-1})^T \overset{?}{=} (V^T)^{-1}$

$A = U \Sigma V^T \qquad A^+ = \text{"}A^{-1}\text{"} = V^{-T} \Sigma^{-1} U^{-1}$

$= V \Sigma^{-1} U^T$



↑ all non-zero (diag.) ↑ pseudoinv.

$\min_x \|Ax - b\|_2 \to x = A^+ b$

$$\begin{pmatrix} 3 & 0 \\ 0 & \frac{1}{2} \\ 0 & 0 \end{pmatrix}^+ = \begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{3} & 0 & 0 \\ 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$A A^{-1} = I$$

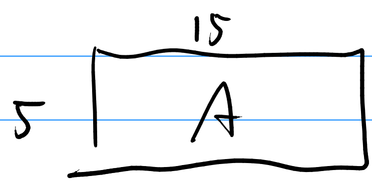
$$U \Sigma V^T A^{-1} = I \quad \Big| \; U^T \cdot$$

$$\underbrace{U^T U}_{id} \Sigma V^T A^{-1} = U^T \quad \Big| \; \Sigma^{-1} \cdot$$

$$\cancel{\Sigma^{-1} \Sigma} \, V^T A^{-1} = \Sigma^{-1} U^T \quad \Big| \; V \cdot$$

$$V V^T A^{-1} = V \Sigma^{-1} U^T$$

Define: $A^+ = V \Sigma^+ U^T$



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$x = b$

look for

$w \approx b$

Looking for $w_1, w_2, \ldots$

$w_1 \cdot$ radius (mean) $+ w_2 \cdot$ (radius (std dev))

$+ w_3 \cdot (\ldots) + \ldots = \begin{cases} +1 & M \\ -1 & B \end{cases}$

Find w s.t. $\|Aw - b\|_2$ is minimized

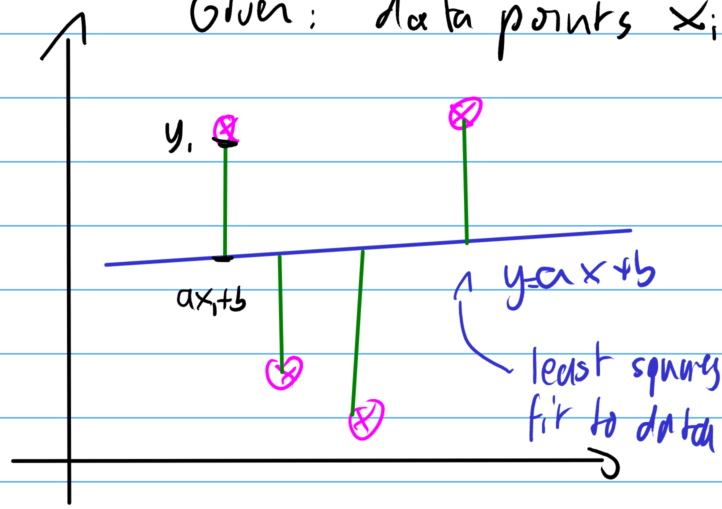
$a_i \cdot w \approx b_i$

$$\left\| \begin{bmatrix} \\ 0 \end{bmatrix} x - Q^T b \right\|_2$$

solve $R_{upper}\, x = (Q^T b)_{upper}$

7 Singular Value Decomposition

Given: data points $\vec{x}_i = (x_i, y_i)$

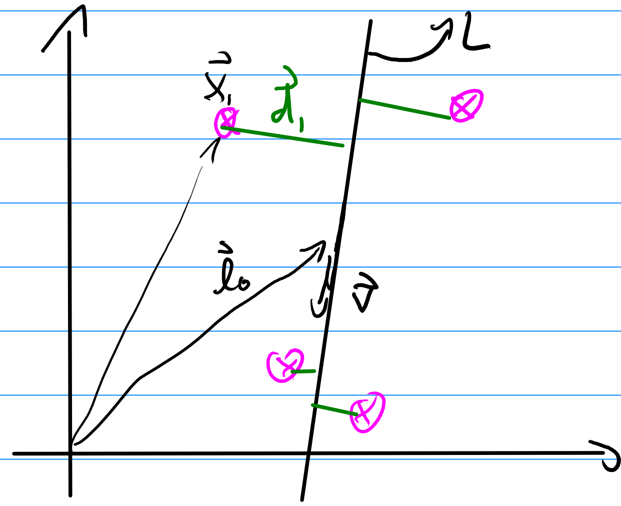


Using least squares

$y_i \approx ax_i + b$

LSQ minimizes

$\sum_i |y_i - (ax_i + b)|^2$



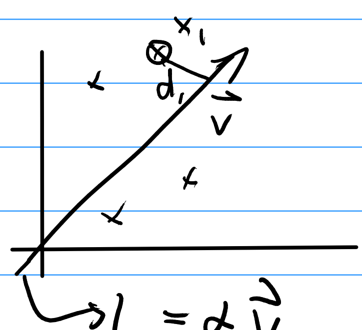
Want to minimize

$\sum_i dist(\vec{x}_i, L)^2$

$\vec{d}_1 =$

$L: \vec{l}_0 + \alpha \vec{v}$

Make a simplifying assumption: $\vec{l}_0 = 0$



$\vec{d}_1 = \vec{x}_1^{\perp v}$

$\vec{x}_1 - \alpha \vec{v}$

↳ $\vec{x}_1^{\parallel \vec{v}}$ $\vec{x}_1^{\perp V}$