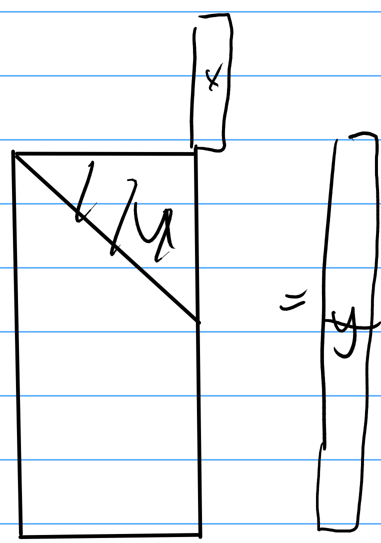
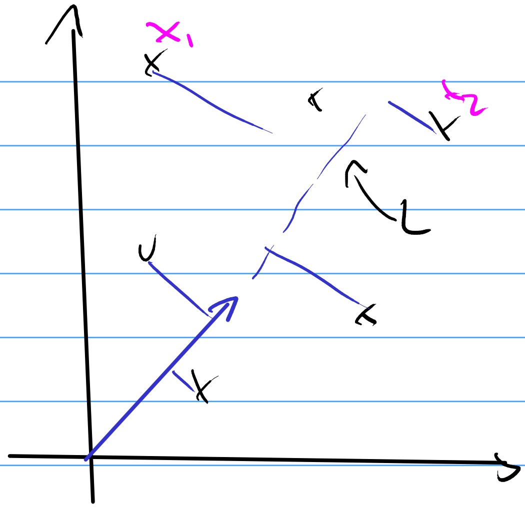
$Ax = b$ $\qquad$ $A = LU$

$LUx = b$ $\qquad$ $Ly = b$

$Ux = y$



$Ux - y =$



$$X = \begin{pmatrix} \text{---} \; x_1 \; \text{---} \\ \vdots \\ \text{---} \; x_n \; \text{---} \end{pmatrix}$$

$$\sum dist(x_i, L)^2 = \| X \|_F^2 - \| X v \|_2^2$$

Frobenius norm

$$A = \begin{pmatrix} a_{11} & a_{1m} \\ a_{n1} & a_{nm} \end{pmatrix}$$

$$\| A \|_F = \sqrt{\sum_{ij} |a_{ij}|^2}$$

Def. Matrix norm

$$\| X \|_2 = \max_{y \neq 0} \frac{\| X y \|}{\| y \|}$$

$$= \max_{y \neq 0} \| X y \| \cdot \left| \frac{1}{\| y \|} \right|$$

$$= \max_{y \neq 0} \left\| X \frac{y}{\| y \|} \right\|$$

$$= \max_{\| y \| = 1} \| X y \|_2$$

$$\min \sum dist^2 = \| X \|_F^2 - \| X \|_2^2$$

① Assume we can find v_1 s.t. $\| X v_1 \|_2 = \| X \|_2$. $\| v_1 \| = 1$

② $\sigma_1 = \| X v_1 \|_2 = \| X \|_2$

③ Find v_2 s.t. $\| X v_2 \|$ is maximized $v_1 \perp v_2$, $\| v_2 \| = 1$

④ $\sigma_2 = \| X v_2 \|_2$

$$\underbrace{\begin{pmatrix}\sigma_1 & & \\ & \ddots & \\ & & \sigma_n\end{pmatrix}}_{\Sigma} \underbrace{\begin{pmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{pmatrix}}_{V}$$

$$A \stackrel{?}{=} U \Sigma V^\dagger$$

columns: right sing. vectors

columns: left singular vecs

$$A = U \Sigma V^\dagger$$

$\rightarrow$ eigenvectors of A

$$\boxed{E}$$

$$A = E D E^{-1}$$

$$V^T v_1$$

$$\begin{pmatrix} - v_1 - \\ - v_2 - \\ \vdots \\ - v_n - \end{pmatrix} v_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \longrightarrow \begin{array}{l} X = U \Sigma V^T \\ X v_1 = \sigma_1 u_1 \end{array}$$

$$\Sigma V^T v_1$$

$$\Sigma \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} \sigma_1 \\ 0 \\ \vdots \\ \end{pmatrix}$$

$$U \underbrace{\Sigma V^T v_1}$$

$$\begin{pmatrix} | & & | \\ u_1 & \cdots & u_n \\ | & & | \end{pmatrix} \begin{pmatrix} \sigma_1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} = \sigma_1 u_1$$

$$\|Ax\|$$

$$\|A(2x)\| = \|2(Ax)\| = |2|\,\|Ax\|$$