$x_0 = \alpha_1 y_1 + \dots + \alpha_n y_n \qquad A y_i = \lambda_i y_i \quad y_i \neq 0$

$$\frac{A^m x_0}{\lambda_1^n} = \alpha_1 \underbrace{\frac{\lambda_1^m}{\lambda_1^n}}_{1} y_1 + \dots + \alpha_n \underbrace{\frac{\lambda_n^m}{\lambda_1^n}}_{\le 1} y_n \qquad \frac{|\lambda_2|}{|\lambda_1|}$$

abs. value of $\left(\frac{|\lambda_n|}{|\lambda_1|}\right)^n$

$\rightarrow Ay = \lambda y \Leftrightarrow (A - \lambda I)y = 0 \Leftrightarrow y \in N(A - \lambda I)$

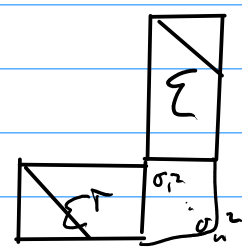
$A = QRQ^T$

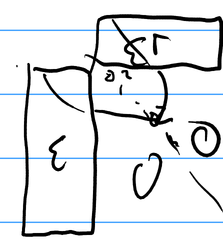
Computing the SVD

$A = U\Sigma V^T \qquad \rightsquigarrow AV = U\Sigma \rightsquigarrow AV\Sigma^{-1} = U$ ($|\cdot V$, $|\cdot \Sigma^{-1}$)

$(A^TA)^T = A^T(A^T)^T = A^TA$

$$A^TA = (U\Sigma V^T)^T U\Sigma V^T = V\Sigma^T \cancel{U^T} U\Sigma V^T = V\Sigma^T\Sigma V^T = V\begin{pmatrix}\sigma_1^2 & & \\ & \ddots & \\ & & \sigma_n^2\end{pmatrix}V^T$$





similarity transform $\rightarrow \sigma_1^2 \dots \sigma_n^2$ are eigvals of A^TA, AA^T

$$AA^T = U\begin{pmatrix}\sigma_1^2 & & \\ & \ddots & \\ & & \sigma_n^2\end{pmatrix}U^T$$

Fact: AA^T and A^TA are symmetric and positive semidefinite
non-neg.

Fact: SPD (symmetric positive definite)
non-neg.

matrices have

- orthogonal eigenvectors
- eigenvalues > 0 ≥ 0

Schur:

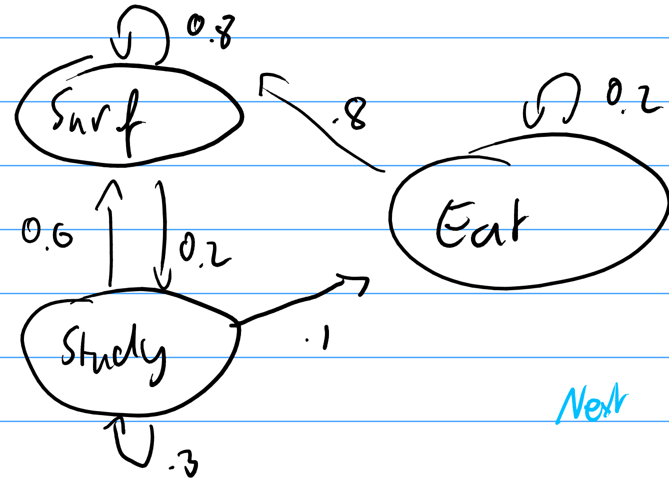
$A = QRQ^T$

Q is orth.

$\Rightarrow Q^T = Q^{-1}$

$A = QRQ^{-1}$

Markov chains



Current state

$$\text{Next}\quad \begin{array}{c|ccc} & \text{surf} & \text{study} & \text{eat} \\ \hline s & .8 & .6 & .8 \\ st & .2 & .3 & 0 \\ e & 0 & .1 & .2 \end{array}$$

Important assumption: Only the most recent state is relevant for determining the probabilities of the next one ("Markov property")

$$\begin{pmatrix} A \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} .8 \\ .2 \\ 0 \end{pmatrix} \leftarrow \text{watching one Andreas}$$

$$\begin{pmatrix} A \end{pmatrix} \left(\frac{1}{2} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) =$$

An interesting thing to find would be the stable distribution x:

$$Ax = x$$

Interpolation

$$f(x) = \alpha_1 \varphi_1(x) + \dots + \alpha_n \varphi_n(x)$$

$$f(x_i) = y_i \quad (i = 1 \dots n)$$

$$\begin{pmatrix} 1 & x_1 & \dots & x_1^{n-1} \\ 1 & & & \\ 1 & & & \\ 1 & x_n & & x_n^{n-1} \end{pmatrix} \begin{pmatrix} \vec{\alpha} \end{pmatrix} = \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$$

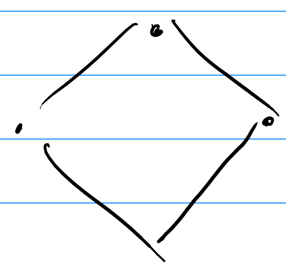
$\uparrow$

monomials: "Vandermonde matrix"

not monomials: generalized Vdm.

$$\begin{array}{cc} 1 & (x - x_1) \\ \left(\begin{matrix} 1 \\ \\ 1 \end{matrix}\right. & \left.\begin{matrix} 0 \\ \\ x_n - x_1 \end{matrix}\right) \end{array}$$

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} \right\|_1 = \sqrt[1]{|x|^1 + |y|^1} = |x| + |y|$$



$3 \cdot 1 - 5x^2$

$3 \cdot 0 - 5 \cdot 2 \cdot x$