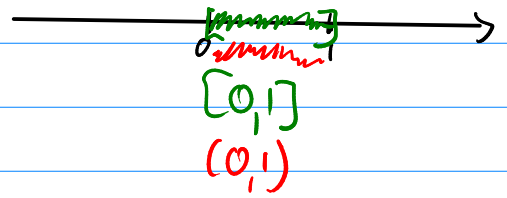Problems with high order poly interp:
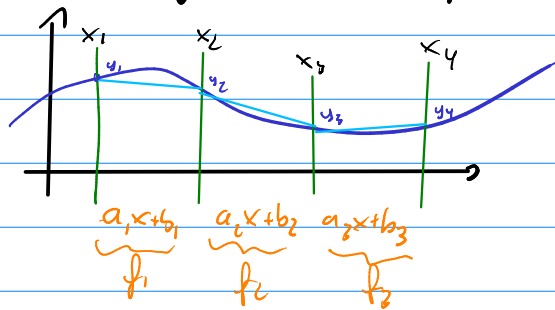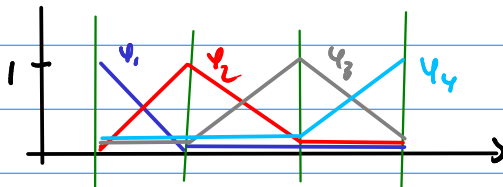
- condition #

- fuctions have to be smooth (= have many derivatives) for it to work
- resolution control is a challenge
- Interp. with full Vandemonde matrices costs $O(n^3)$
  → Newton interpolation V triangular $O(n^2)$
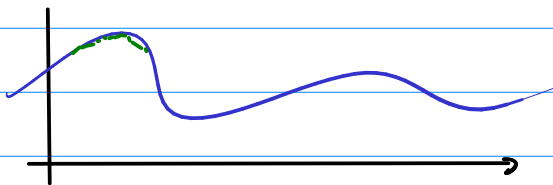 → Lagrange interpolation V diagonal $O(n)$
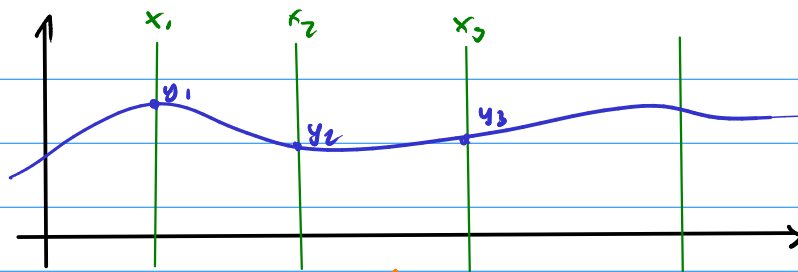
Piecewise polynomial interpolation

$a_1 x + b_1$ $a_2 x + b_2$ $a_3 x + b_3$

f_1 f_2 f_3

$f_1(x_1) = y_1$ $f_2(x_2) = y_2$

$f_1(x_2) = y_2$ $f_2(x_3) = y_3$

"hat function basis"

$$V = \begin{pmatrix} \varphi_1(x_1) & & \varphi_4(x_1) \\ & & \\ \varphi_1(x_4) & & \varphi_4(x_4) \end{pmatrix} = Id.$$

x_1 x_2 x_3

y_1 y_2 y_3

12 coeff

$a_1x^3 + b_1x^2 + c_1x + d_1$ $a_3x^3 + b_3x^2 + c_3x + d$

f_1

$a_2x^3 + b_2x^2 + c_2x + d_2$

f_2

6 equations

$f_1(x_1) = y_1$ $f_2(x_2) = y_2$ $f_3(x_3) = y_3$

$f_1(x_2) = y_2$ $f_2(x_3) = y_3$ $f_3(x_4) = y_4$

$f_2'(x_2) = f_3'(x_2)$

" Splines "

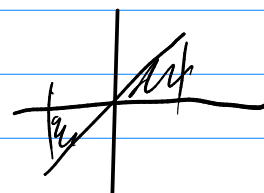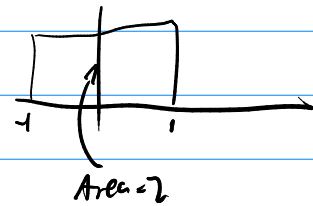" cable Splines "

$\longrightarrow$ HW

Orthogonal polymials

$\underbrace{3x^3 + 4x - 7}_{p}$ orth. to $\underbrace{5x - 2}_{q}$?

$$(p,q) = \int_0^1 p(x) q(x) dx$$

$$\int_{-1}^1 1 \cdot 1 \, dx$$

Area = 2

Fourier

$$\underbrace{\underbrace{\cos(0x)}_{1}, \sin(1\cdot x), \cos(1x), \sin(2x), \dots}$$

$$y = T^{-1}\underbrace{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}_{x}$$

$$T^{-1}(A-15I)T\ y$$

$$= T^{-1}(A-15I)\ \cancel{T}\ T^{-1}x$$

$$= T^{-1}(A-15I)\ x$$

$$= T^{-1}(Ax - 15Ix)$$

$$= T^{-1}(12x - 15x)$$

$$= (-3)T^{-1}x = (-3)y$$

$$T^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 0 & 1 \end{pmatrix}$$

$$T^{-1}x = \swarrow \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$