Choosing nodes for polynomial interpolation

Observation Discussed choice of basis

Also have choice of nodes

So far: only used equispaced nodes

Observation:

Best nodes set on [-1,1]: "Chebyshev nodes"

$$x_k = \cos\left(\frac{2k-1}{2n}\pi\right) \qquad k = 1 \ldots n$$

⑤ Sines and Cosines ("Fourier Basis")

Idea: Use sin(nx) and cos(nx) as a basis for a number of values of n

What's the basis? $\cancel{\sin(0x)}$ $\cos(0x)$ $\sin(1x)$ $\cos(1x)$ $\sin(2x)$ $\cos(2x)$

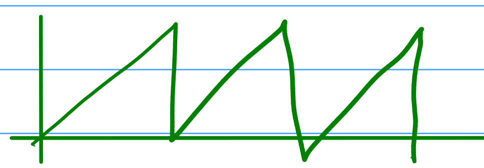
What to use as points? $[0, 2\pi]$

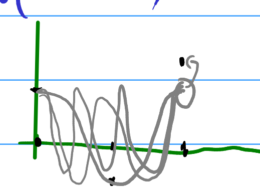
equispaced

$2\pi\cdot\frac{0}{5},\quad 2\pi\cdot\frac{1}{5},\quad 2\pi\cdot\frac{2}{5},\ \dots\quad 2\pi\cdot\frac{4}{5},\quad \cancel{2\pi\cdot\frac{5}{5}}$

Write down a Vandermonde matrix for up to n=2

$$
\begin{matrix}
\cos(0\cdot 2\pi\frac{0}{5}) & \sin(1\cdot 2\pi\frac{0}{5}) & \cos(1\cdot 2\pi\frac{0}{5}) & \sin(2\cdot 2\pi\frac{0}{5}) & \cos(2\cdot 2\pi\frac{0}{5}) \\
\cos(0\cdot\quad) & \sin(1\cdot\quad) & \cos(1\cdot\quad) & \sin(2\cdot\quad) & \cos(2\cdot\quad) \\
\cos(0\cdot\quad) & \sin(1\cdot\quad) & \cos(1\cdot\quad) & \sin(2\cdot\quad) & \cos(2\cdot\quad) \\
\cos(0\cdot\quad) & \sin(1\cdot\quad) & \cos(1\cdot\quad) & \sin(2\cdot\quad) & \cos(2\cdot\quad) \\
\cos(0\cdot\quad) & \sin(1\cdot\quad) & \cos(1\cdot\quad) & \sin(2\cdot\quad) & \cos(2\cdot\quad)
\end{matrix}
$$





Observations:

- works best for periodic f.
- V is almost orthogonal

Why care about the Fourier basis?