Convergence Rates of Iterative Procedures

Consider the "error" in the kth step of some method:

$e_k = |x - x_k|$ (true answer / guess in the kth step)

What could the error be in the next step, relative to e_k ?

$$e_{k+1} = \frac{e_k}{2}$$

Generally, error behavior like this is called "linear convergence" ("order 1"):

$$e_{k+1} \leq C \cdot e_k^1 \qquad \text{with } 0 \leq C < 1$$

Generally, error behavior like this is called "quadratic convergence" ("order 2"):

$$e_{k+1} \leq C \cdot e_k^2 \qquad \text{with } 0 \leq C < 1$$

Generally, error behavior like this is called "cubic convergence" ("order 3"):

$$e_{k+1} \leq C \cdot e_k^3$$

(... and so on) Which of these is fastest? cubic

Rewrite this so that the constant stands on its own, for a general order q :

$$C \approx \frac{e_{k+1}}{e_k^q}$$

Do not confuse this with "q-th order" convergence $\sim C \cdot h^q$ for a mesh width h!