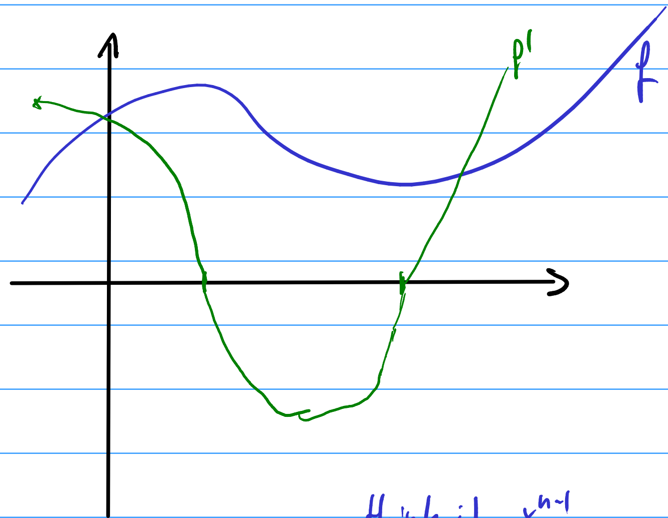
Taking derivatives



think 1 $\qquad x^{n-1}$

$$f(x_i) = \tilde{f}(x_i) \quad i = 1 \dots n$$

Have: $\tilde{f}(x) = \alpha_1 \varphi_1(x) + \cdots + \alpha_n \varphi_n(x)$

Want: $\tilde{f}'(x) = \left(\alpha_1 \varphi_1(x) + \cdots + \alpha_n \varphi_n(x)\right)'$

$$= \alpha_1 \varphi_1'(x) + \cdots + \alpha_n \varphi_n'(x)$$

Have: $f(x_i)$

Want: approx $f'(x_i) \leftarrow$ hard to get

$\tilde{f}'(x_i) \leftarrow$ easy to get

① $V\vec{\alpha} = \vec{f} \rightsquigarrow \vec{\alpha} = V^{-1}\vec{f}$ $\qquad \vec{\alpha} = \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}$ $\qquad \vec{f} = \begin{pmatrix} f(x_1) \\ \vdots \\ f(x_n) \end{pmatrix}$

② $V = \begin{pmatrix} \varphi_1(x_1) & & \varphi_n(x_1) \\ & & \\ \varphi_1(x_n) & & \varphi_n(x_n) \end{pmatrix}$

$$V' = \begin{pmatrix} \varphi_1'(x_1) & \cdots & \varphi_n'(x_1) \\ \vdots & & \vdots \\ \varphi_1'(x_n) & \cdots & \varphi_n'(x_n) \end{pmatrix}$$

$$V'\alpha = \begin{pmatrix} \alpha_1\varphi_1'(x_1) + \cdots + \alpha_n \varphi_n'(x_1) \\ \vdots \\ \alpha_1 \varphi_1'(x_n) + \cdots + \alpha_n \varphi_n'(x_n) \end{pmatrix} = \underbrace{\begin{pmatrix} \tilde{f}'(x_1) \\ \vdots \\ \tilde{f}'(x_n) \end{pmatrix}}_{\tilde{f}'}$$

$$\tilde{f}' = \underbrace{V'V^{-1}} \vec{f}$$