(8) Eigenvalues

$A \in \mathbb{R}^{n \times n}$

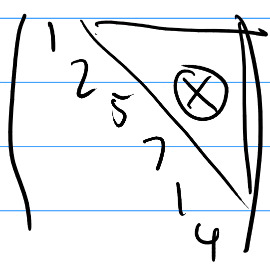
x is an eigenvector of A if:

- $Ax = \lambda x$ for some λ
- $x \neq 0$

λ is called an eigenvalue

$\iff$ $(A - \lambda I)x = 0$ has a solution $x \neq 0$

$\iff$ $N(A - \lambda I) \neq \{0\}$



What matrix transformations do to eigenvalues

Suppose $Ax = \lambda x$ with $x \neq 0$.

- "Scaling" (βA) $\rightsquigarrow$ $(\beta A)x = \beta\lambda x$

- "Shift" $(A - \sigma I)x \rightsquigarrow Ax - \sigma x$
  $= \lambda x - \sigma x$
  $= (\lambda - \sigma)x$

- "Power" $A^k = \underbrace{AAA\cdots A}_{k\text{ times}} \rightarrow A^k x = A^{k-1}\lambda x$
  $= \lambda^k x$

- "Inverse" A^{-1} $\rightsquigarrow$ $Ax = \lambda x \quad | \cdot A^{-1}$
  $x = \lambda A^{-1}x$
  $\lambda^{-1}x = A^{-1}x$

- "Similarity" $T^{-1}AT \rightsquigarrow$ $\quad y := T^{-1}x$

$$(T^{-1}AT)y$$

$$= T^{-1}A\underbrace{TT^{-1}}_{\text{Id}}x$$

$$= T^{-1}Ax$$

$$= \lambda T^{-1}x = \lambda y$$

$\rightsquigarrow$ similarity transform preserves eigenvalues

A, B are called similar if there is

a T so that $B = T^{-1}AT$

We'll call A diagonalizable if

it is similar to a diagonal matrix.