(6) Orthogonality

Definition An inner product is a function of two vector arguments (v,w) that returns a scalar for v,w in a vector space V and satisfies

linearity in first argument
$$\begin{cases} (\alpha x, y) = \alpha(x,y) \\ (x+y, z) = (x,z) + (y,z) \end{cases}$$
symmetry $\{ (x,y) = (y,x)$

pos. semidef. $(x,x) \geq 0$

for $x,y,z \in V$

(pos. definite) $(x,x) = 0 \Leftrightarrow x = 0$

Example
$$\begin{pmatrix} 1 \\ 2 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 10 \\ 20 \\ 1 \end{pmatrix} = 1 \cdot 10 + 2 \cdot 20 + 5 \cdot 1$$

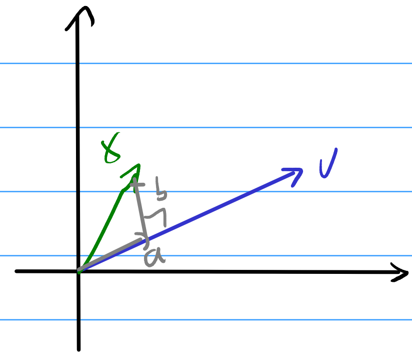
one example of an inner product on $\mathbb{R}^3$

Definition: Two vectors x,y are orthogonal or perpendicular

if $(x,y) = 0 \Leftrightarrow: x \perp y$

- $\|x\| := \sqrt{(x,x)}$

Making two vectors orthogonal to each other



$$x = \overset{\parallel v}{a} + \overset{\perp v}{b}$$

$$a = \alpha \cdot v$$

$$b \perp v$$

$$0 = (b, v) \overset{b = x - a}{=} (x - a, v)$$

$$= (x - \alpha v, v)$$

$$= (x, v) - \alpha (v, v)$$

$$\alpha = \frac{(x, v)}{(v, v)} v$$

$$a = x^{\parallel v} = \frac{(x, v)}{(v, v)} v$$

$$b = x^{\perp v} = x - a = x - \frac{(x, v)}{(v, v)} v$$