Interpolation error

$|f(x) - \tilde{f}(x)| \leq ?$

$\hat{f}(x) = a_0 1 + a_1 x + \cdots a_n x^{n-1}$

$\tilde{f}(x_i) = f(x_i) \quad i = 1 \ldots n$

$$V\left(\vec{a}\right) = \begin{pmatrix} f(x_1) \\ \vdots \\ f(x_n) \end{pmatrix}$$

$$|f(x) - \tilde{f}(x)| \leq C \cdot h^{n+1}$$

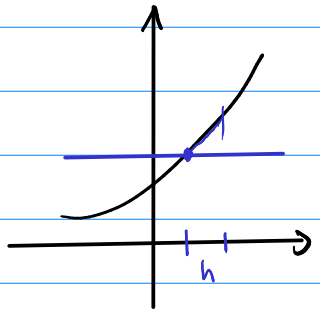
"true function" "interpolant"

C allowed to depend on n, f

n : highest poly degree

h : length of the interval

Example: $n = 0$



Suppose I've got an interpolant for some function of degree 2 (using quadratic functions at the most)

$\leftarrow n = 2$

$$\text{Error}(h) \approx C \cdot h^3$$

$$\text{Error}\left(\frac{h}{2}\right) \approx C \cdot \left(\frac{h}{2}\right)^3 = \frac{1}{8} \cdot C \cdot h^3$$

$$= \frac{1}{8} \cdot \text{Error}(h)$$

$$\text{Error} \leq C \cdot h^p$$ ← "pth order convergent"