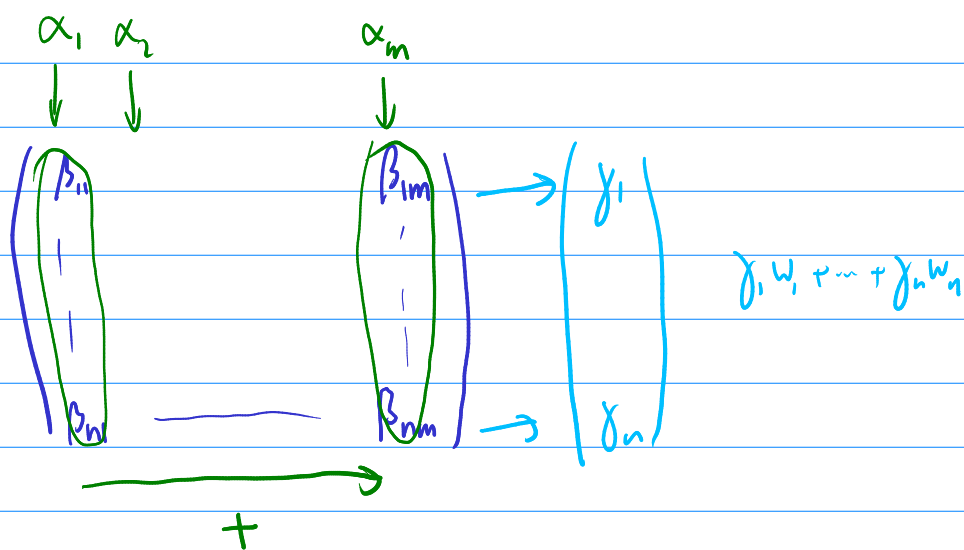


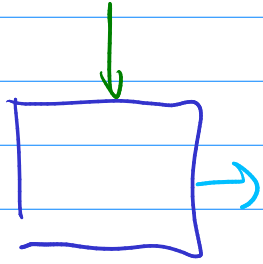
"input" V : spanned by generators v_1, v_2, \dots, v_m

"output" W : spanned by generators w_1, w_2, \dots, w_n

$f: V \rightarrow W$ linear



$$\begin{pmatrix} \beta_{11} & \beta_{1m} \\ \vdots & \vdots \\ \beta_{n1} & \beta_{nm} \end{pmatrix} \cdot \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{pmatrix} = \begin{pmatrix} \delta_1 \\ \vdots \\ \delta_n \end{pmatrix}$$



$$\begin{array}{c}
 \boxed{0 \ 1 \ 0} \xleftarrow{f(v_2)} \\
 \left(\begin{array}{ccc|c}
 3 & 4 & 3 & \rightarrow 4 \\
 2 & 7 & 1 & \rightarrow 7 \\
 4 & \textcircled{0} & 1 & \rightarrow \underline{\underline{0}}
 \end{array} \right)
 \end{array}$$

$$\begin{array}{c}
 (0 \ 0 \ 1) \quad f(v_3) \\
 \left(\begin{array}{ccc|c}
 \otimes & \otimes & \otimes & \rightarrow \beta_{13} \\
 \otimes & \otimes & \otimes & \rightarrow 0 \\
 \otimes & \otimes & \otimes & \rightarrow 0
 \end{array} \right) \quad \rightarrow \quad \left(\begin{array}{c} \beta_{13} \\ 0 \\ 0 \end{array} \right)
 \end{array}
 \quad \rightarrow \quad f(v_3) = \beta_{13} \cdot w_1$$

$$\begin{array}{c}
 (\alpha_1 \ \alpha_2 \ \alpha_3) \quad f(\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3) \\
 \left(\begin{array}{ccc|c}
 \otimes & 0 & \otimes & \rightarrow \dots \\
 \beta_{21} & 0 & 0 & \rightarrow \beta_{21} \alpha_1 \\
 \otimes & \otimes & 0 & \rightarrow \dots
 \end{array} \right)
 \end{array}$$

$$f(v_3) = w_1$$

$$f(v_2) = w_3$$

$$\begin{matrix} (0 & 0 & 1) \\ \left(\begin{array}{c|c} \boxed{?} & \begin{array}{c} 1 \\ 0 \\ 0 \end{array} \end{array} \right) \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \end{matrix}$$

$$\begin{matrix} (0 & 1 & 0) \\ \left(\begin{array}{c|c} \boxed{?} & \boxed{?} \\ \hline 0 & 1 \end{array} \right) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \end{matrix}$$