

$$\begin{aligned} V: & v_1, v_2, v_3, \dots, v_m \in V \\ W: & w_1, w_2, w_3, \dots, w_n \in W \end{aligned}$$

$$x = \alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_m v_m$$

$$f(x) = \alpha_1 f(v_1) + \alpha_2 f(v_2) + \dots + \alpha_m f(v_m)$$

$$\begin{array}{ccc} \downarrow & & \downarrow \\ \beta_{11}v_1 + \dots + \beta_{m1}v_m & & \beta_{1n}w_1 + \dots + \beta_{mn}w_n \\ \begin{array}{c} \alpha_1 \\ \downarrow \\ \beta_{11} \\ \vdots \\ \beta_{m1} \end{array} & \begin{array}{c} \alpha_2 \\ \downarrow \\ \vdots \\ \beta_{m2} \end{array} & \begin{array}{c} \alpha_m \\ \downarrow \\ \beta_{1m} \\ \vdots \\ \beta_{mm} \end{array} \\ \left. \begin{array}{c} \vdots \\ \vdots \\ \vdots \end{array} \right\} \begin{array}{l} \rightarrow \gamma_1 \\ \vdots \\ \rightarrow \gamma_n \end{array} \end{array}$$

$$f(x) = \gamma_1 w_1 + \dots + \gamma_n w_n$$

$$\begin{array}{ccc} \left( \begin{array}{c} \beta_{11} \\ \vdots \\ \beta_{m1} \end{array} \right) & \left( \begin{array}{c} \beta_{1m} \\ \vdots \\ \beta_{mm} \end{array} \right) & \begin{array}{l} \left( \begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_m \end{array} \right) \\ = \\ \left( \begin{array}{c} \gamma_1 \\ \vdots \\ \gamma_n \end{array} \right) \end{array} \end{array}$$

$$\begin{array}{ccc} f: V \rightarrow W & g: W \rightarrow X & \text{both linear} \\ \downarrow & \downarrow & \\ f(x) = Ax & g(x) = Bx & \end{array}$$

$$g(f(x)) = B(A \cdot x)$$