(11) **Solving nonlinear equations**

$ax = b$

Have: $f: \mathbb{R} \to \mathbb{R}$ function $\qquad Ax = b$

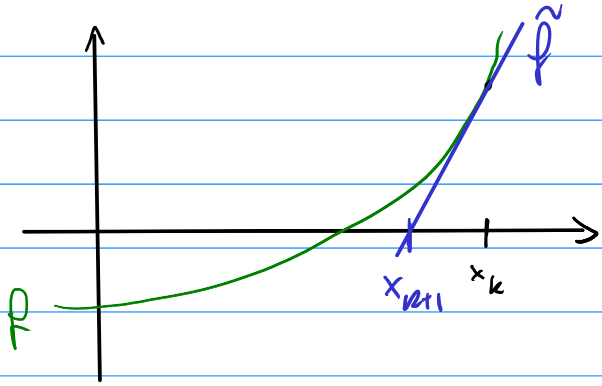
Want: x such that $f(x) = y$ $\qquad f(x) = x^3 - x + 1 \qquad f(x) = 15$

Rewrite the problem so that we only need $g(x) = 0$. (i.e. no explicit right-hand side)

$$g(x) = f(x) - y \qquad f(x) = y \iff g(x) = 0$$

Newton's method

Suppose x_k is our current guess of the zero.



Idea: Build a solvable approximate version of f using $f(x_k)$ and $f'(x_k)$

$$\tilde{f}(x_k+h) = f(x_k) + h \cdot f'(x_k)$$

Find the zero of the approximate version.

$$0 = \tilde{f}(x_k) = f(x_k) + h \cdot f'(x_k)$$

$$-f(x_k) = h f'(x_k)$$

$$-\frac{f(x_k)}{f'(x_k)} = h$$

$$x_{k+1} = x_k + h = x_k - \frac{f(x_k)}{f'(x_k)}$$

This is called Newton's method.

$$f(x) = x^3 - x + 1$$
$$f'(x) = 3x^2 - 1$$

$$x_{k+1} = x_k - \frac{x_k^3 - x_k + 1}{3x_k^2 - 1}$$