$\|x\| = \sqrt{x_0^2 + x_1^2 + x_2^2}$

$x = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \end{pmatrix}$

Definition A norm is a function $\|\cdot\|$ (input) from a vector space V into the real numbers that satisfies:

- $\|x\| \geq 0$
- $\|x+y\| \leq \|x\| + \|y\|$ (triangle inequality)
- $\|\alpha x\| = |\alpha| \, \|x\|$
- $\|x\| = 0 \iff x = 0$

Examples Let $p \geq 1$. Then we define for a vector x with coordinates $(x_1, \dots, x_n)$

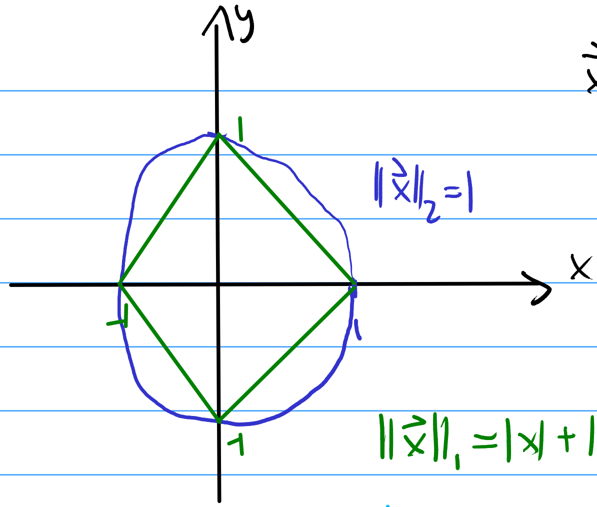
$$\|x\|_p = \sqrt[p]{|x_1|^p + |x_2|^p + \dots + |x_n|^p}$$

$$\|x\|_2 = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$$

$$\|x\|_1 = |x_1| + \dots + |x_n|$$

$$\|x\|_\infty \text{"="} \sqrt[\infty]{|x_1|^\infty + |x_2|^\infty + \dots + |x_n|^\infty}$$
$$= \max(|x_1|, |x_2|, \dots, |x_n|)$$

$$\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$$



$\|\vec{x}\|_2 = 1$

$\|\vec{x}\|_1 = |x| + |y| = 1$

$\|x\|_\infty = 1$?

linear

$$\|f\| = \max_{x \neq 0} \frac{\|f(x)\|}{\|x\|}.$$