

Power iteration

A diagonalizable $n \times n$ - have x_1, \dots, x_n eigenvectors l.i.
with $\lambda_1, \dots, \lambda_n$ eigenvalues

$$|\lambda_1| \geq |\lambda_2| \dots \geq |\lambda_n|$$

$$x = \alpha_1 x_1 + \dots + \alpha_n x_n$$

$$\begin{aligned} Ax &= \alpha_1 Ax_1 + \dots + \alpha_n Ax_n \\ &= \alpha_1 \lambda_1 x_1 + \dots + \alpha_n \lambda_n x_n \end{aligned}$$

$$A^{20,000} x = A \cdot A \cdot A \dots Ax$$

$$= \alpha_1 \lambda_1^{20,000} x_1 + \dots + \alpha_n \lambda_n^{20,000} x_n$$

$$\frac{A^{20,000} x}{\lambda_1^{20,000}} = \alpha_1 x_1 + \alpha_2 \underbrace{\frac{\lambda_2^{20,000}}{\lambda_1^{20,000}}}_{\substack{|\cdot| \leq 1 \\ = \left(\frac{\lambda_2}{\lambda_1}\right)^{20k}}} x_2 + \dots + \alpha_n \underbrace{\frac{\lambda_n^{20,000}}{\lambda_1^{20,000}}}_{\substack{\downarrow \\ = \left(\frac{\lambda_n}{\lambda_1}\right)^{20k}}} x_n$$