$v_1, v_2, \dots, v_n$ linearly dep. if

$\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_n v_n = 0$ & at least one $\alpha_i \neq 0$.

$$\begin{array}{c} \begin{array}{cccccc} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 & & \alpha_n \end{array} \\ \underbrace{\begin{bmatrix} | & | & | & | & & | \\ v_1 & v_2 & v_3 & v_4 & \cdots & v_n \\ | & | & | & | & & | \end{bmatrix}}_{A} \end{array} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}$$

If there's $\vec{\alpha} \neq 0$ s.t. $A\vec{\alpha} = \vec{0}$, then (v_i) linearly dependent.

kernel

Definition: Nullspace of a linear function

$$N(f) = \{ \vec{x} : f(\vec{x}) = 0 \}$$

ker(f)

Similarly: $N(A) \leftarrow \ker(A)$

$\dim(N(A)) \leftarrow$ "nullity"

$$A = \begin{pmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_n \\ | & | & & | \end{pmatrix}$$

$\text{span}(v_1, v_2, \ldots, v_n) =$ "<u>column space</u>"

<u>column rank</u>$(A) = \dim(\text{column space})$

$$A = \begin{pmatrix} — & \bar{v}_1 & — \\ — & \bar{v}_2 & — \\ & \vdots & \\ — & \bar{v}_m & — \end{pmatrix}$$

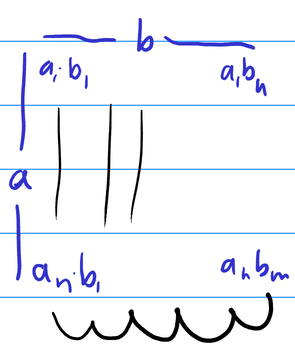
$\text{span}(\bar{v}_1, \bar{v}_2, \ldots, \bar{v}_m) =$ "<u>row space</u>"

<u>row rank</u>$(A) = \dim(\text{row space})$

$$\text{col rank} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} = 2$$

$$\text{row rank} \begin{pmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} = 2$$

<u>Fact:</u> $\cancel{\text{col}}\ \underline{\text{rank}}(A) = \cancel{\text{row}}\ \underline{\text{rank}}(A)$



$\leftarrow$ outer product

lin indep?

rank(outer product) $= 1$

- $\dim N(A) + \operatorname{rank}(A) = \#$ columns

A repr. $f: \underset{\dim n}{V} \to \underset{\dim m}{W}$ $\quad {}^{m \times n}$

full rank $\Rightarrow \operatorname{rank}(A) = n$