Numerical Methods (CS 357)

Worksheet

Problem 1. Value of the condition number

Consider the matrix

$$A = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}.$$

What's the value of the 2-norm-based condition number of A?

Problem 2. Nullspace Finding

Given a LU factorization $PA = LU$ of a matrix A, we know that the nullspace is preserved by this facatorization as $N(PA) = N(U)$. Which of the following are true statements?

(A) The nullspace of A can be "read off" from U with little (at most linear in n) computational work.

(B) Having an LU factorization of A does not help significantly with computing the nullspace of A.

(C) Computing the nullspace is inherently brittle because of rounding error.

(D) Matrices in echelon form do not have a nullspace.

Problem 3. Nullspace Finding II

What's the nullspace of

$$U^T = \begin{bmatrix} * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & 0 & 0 \end{bmatrix}$$

irrespective of the values of the $*$ entries?

(A) Unable to determine

(B) $N(U^T) = \{[0, 0, 0, 1, 0]^T, [0, 0, 0, 0, 1]^T\}$

(C) $N(U^T) = \text{span}\{[0, 0, 0, 0, 1, 0]^T, [0, 0, 0, 0, 0, 1]^T\}$

(D) $N(U^T) = \text{span}\{[0, 0, 0, 1, 0]^T, [0, 0, 0, 0, 1]^T\}$

(E) $N(U^T) = \text{span}\{[0, 0, 0, 1, 1]^T\}$