Problem 1. Value of the condition number

Consider the matrix

\[
A = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}.
\]

What’s the value of the 2-norm-based condition number of \( A \)?

Problem 2. Nullspace Finding

Given a LU factorization \( PA = LU \) of a matrix \( A \), we know that the nullspace is preserved by this factorization as \( N(PA) = N(U) \). Which of the following are true statements?

(A) The nullspace of \( A \) can be “read off” from \( U \) with little (at most linear in \( n \)) computational work.

(B) Having an LU factorization of \( A \) does not help significantly with computing the nullspace of \( A \).

(C) Computing the nullspace is inherently brittle because of rounding error.

(D) Matrices in echelon form do not have a nullspace.

Problem 3. Nullspace Finding II

What’s the nullspace of

\[
U^T = \begin{bmatrix}
* & 0 & 0 & 0 & 0 \\
* & * & 0 & 0 & 0 \\
* & * & * & 0 & 0 \\
* & * & * & * & 0 \\
* & * & * & * & 0
\end{bmatrix}
\]

irrespective of the values of the \( * \) entries?
(A) Unable to determine 

(B) $N(U^T) = \{ [0, 0, 0, 1]^T, [0, 0, 0, 1]^T \}$

(C) $N(U^T) = \text{span}\{ [0, 0, 0, 1]^T, [0, 0, 0, 1]^T \}$

(D) $N(U^T) = \text{span}\{ [0, 0, 1, 0]^T, [0, 0, 0, 1]^T \}$

(E) $N(U^T) = \text{span}\{ [0, 0, 1, 1]^T \}$