Numerical Methods (CS 357)

Worksheet

Problem 1. Value of the condition number

Consider the matrix

$$A = \begin{pmatrix} 5 & 0 \\ 0 & 20 \end{pmatrix}.$$

What's the value of the 2-norm-based condition number of A?

Problem 2. Problems with Rank Finding

Which of the following is a problem with finding the rank of a number of vectors computationally using pivoted Gaussian elimination?

(A) Infeasibly expensive (in terms of computational work)

(B) May break down if near-zeros occur in input

(C) Answer brittle/poorly defined due to rounding errors

(D) Only works for 'square' sets of vectors, i.e. sets of n vectors of length n

Problem 3. Computational expense of solving many linear systems

Suppose you have both the inverse A^{-1} and a PLU factorization of an $n \times n$ matrix. What is true about the computational expense of finding the solution of k linear systems $Ax_i = b_i$ $(i = 1, \ldots, k)$ using both of these methods?

(A) Using the inverse is asymptotically cheaper (n^2 vs n^3)

(B) Asymptotically, the two methods have the same computational cost (n^2 and n^2)

(C) Asymptotically, the two methods have the same computational cost (n^3 and n^3)

(D) Using LU is asymptotically cheaper (n^3 vs n^2)

Problem 4. Nullspace Finding

Given a LU factorization $PA = LU$ of a matrix A, we know that the nullspace is preserved by this facatorization as $N(PA) = N(U)$. Which of the following are true statements?

(A) The nullspace of A can be "read off" from U with little (at most linear in n) computational work.

(B) Having an LU factorization of A does not help significantly with computing the nullspace of A.

(C) Computing the nullspace is inherently brittle because of rounding error.

(D) Matrices in echelon form do not have a nullspace.