

Numerical Methods (CS 357)
Worksheet

Problem 1. Back-substitution

Consider an $n \times n$ upper triangular matrix. How many operations are carried out in back-substitution?

- (A) Independent of n
- (B) Proportional to n
- (C) Proportional to n^2
- (D) Proportional to n^3

Problem 2. Rank and Nullspace

A matrix of size 10×7 has a row space of dimension 5. What is the dimension of its nullspace?

- (A) 2
- (B) 7
- (C) 10
- (D) 0
- (E) 5

Problem 3. Elimination matrices

Consider the elimination matrices

$$A = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & 2 & 1 & \\ & -4 & & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & -2 & 1 & \\ & 4 & & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & 5 & 1 \end{pmatrix}.$$

What is ABC ?

$$(A) \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & 5 & 1 \end{pmatrix}$$

$$(B) \begin{pmatrix} 1 & & & \\ & 1 & & \\ & -4 & 1 & \\ & 8 & & 1 \end{pmatrix}$$

$$(C) \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$(D) \begin{pmatrix} 1 & & & \\ & 1 & & \\ & 2 & 1 & \\ & 2 & & 1 \end{pmatrix}$$

Problem 4. Invariants of LU

Suppose you have an LU factorization $PA = LU$.

Is $\text{rowspace}(U) = \text{rowspace}(PA)$?

(A) Yes

(B) No

Problem 5. Invariants of LU

Suppose you have an LU factorization $PA = LU$.

Is $N(U) = N(PA)$?

(The original version of this question specified got the answer wrong.)

(A) Yes

(B) No