Problem 1. Least-squares residual

Given a QR factorization $A = QR$ with $Q = I$, what is the **square of the 2-norm** of the residual of solving the least-squares problem $Ax \approx b$ going to be if

$$b = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 2 \end{bmatrix}, \qquad R = \begin{bmatrix} 5 & 2 \\ 0 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}?$$

Problem 2. Least-squares residual

Let x be the solution to the linear least squares problem $Ax \simeq b$, where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}.$$

Let $r = b - Ax$ be the corresponding residual vector. Which of the following vectors is a possible value for r?

(A) $\begin{bmatrix} 3 \\ 0 \\ -4 \\ 3 \end{bmatrix}$

(B) $\begin{bmatrix} 0 \\ 20 \\ -10 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 \\ 2 \\ 3 \\ -4 \end{bmatrix}$

(D) $\begin{bmatrix} 1 \\ 2 \\ 4 \\ -1 \end{bmatrix}$

Problem 3. Least-squares residual

Given a QR factorization $A = QR$ with $Q = I$, what is the last component of the least-squares solution x in $\min \|Ax - b\|_2$ going to be if

$$
b = \begin{bmatrix} 3 \\ 1 \\ 4 \\ 2 \end{bmatrix}, \qquad R = \begin{bmatrix} 5 & 2 \\ 0 & 5 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} ?
$$