Numerical Methods (CS 357)

Worksheet

Problem 1. Linearly independent vectors

Consider this set S of vectors

$$\begin{bmatrix}1\\0\\0\\1\end{bmatrix}, \quad \begin{bmatrix}-1\\0\\0\\-1\end{bmatrix}, \quad \begin{bmatrix}0\\1\\0\\1\end{bmatrix}.$$

Which vector can you add to S and have the resulting set of vectors be linearly independent?

(A) None

(B) $\begin{bmatrix}0\\0\\1\\1\end{bmatrix}$

(C) $\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$

(D) $\begin{bmatrix}-1\\0\\0\\1\end{bmatrix}$

Problem 2. Guessing Dimensions

Suppose I have three vectors v_1, v_2, v_3.
What are possible values of

$$\dim(\text{span}(\{v_1 + v_2, -v_1 - v_2, 0v_3\}))?$$

(A) 0 or 1

(B) 0, 1, or 3

(C) 1 or 3

(D) 0 or 2

(E) 0, 1, or 2

Problem 3. Building a basis

Suppose I have a basis of $\mathbb{R}^3$. Which of the following procedures reliably yields a basis of $\mathbb{R}^4$? (Several choices could be correct.)

(A) Add a one as the last coordinate to each vector, e.g. taking $(3, 4, 7)$, to $(3, 4, 7, 1)$.

(B) Add a one as the last coordinate to each vector, and add $(0, 0, 0, 1)$ as an additional vector.

(C) Add a one as the last coordinate to each vector, and add $(1, 0, 0, 0)$ as an additional vector.

(D) Add a zero as the last coordinate to each vector, e.g. taking $(3, 4, 7)$, to $(3, 4, 7, 0)$.

Problem 4. Dimension of the Space of Images

What's the dimension of the space of all 100×100 gray scale images?

(A) 10,000

(B) 100

(C) However many gray scale levels there are

(D) 0