Linear Algebra and Computation

What is a vector?

$$\begin{pmatrix} 3 \\ 7 \\ 15 \end{pmatrix}$$

The real question is what we want from a vector.

What do we want to do with it?

$v, w \in V$ vector space

α a real or complex number

$\rightsquigarrow$ $v + w \in V$

$\underset{\text{scalar}}{\alpha}\, v$

$1 \cdot v = (-2+3) \cdot v = -2 \cdot v + 3 \cdot v$

What on earth are you talking about?

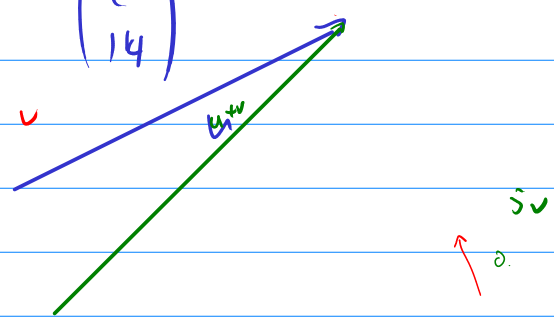
```
interface Vector {

    Vector add(Vector x, Vector y)
    Vector scalar_multiply(double x, Vector y)

}
```

So, is a plain old number a vector, too?

$$\begin{pmatrix} 2 \\ 14 \end{pmatrix}$$

What else can be viewed a vector?

$$\begin{pmatrix} 2 \\ 14 \end{pmatrix}$$



So how are all these things similar?

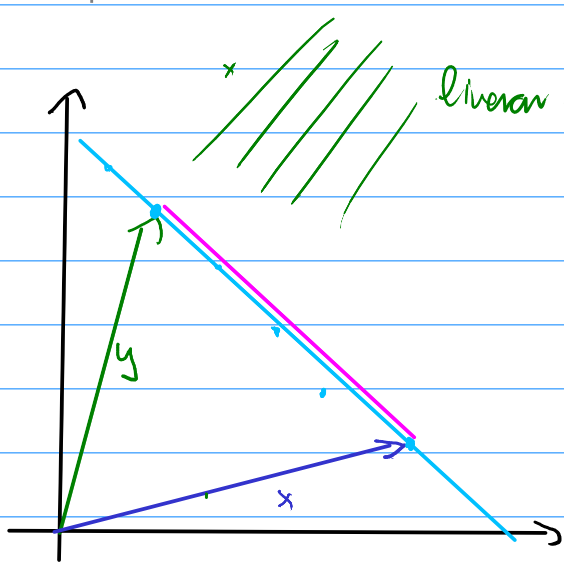
+ .

What are some useful things one can do in a vector space?

linear combinations	coeff. $\alpha \cdot \vec{x} + \beta \cdot \vec{y} + \gamma \cdot \vec{z}$
affine combinations	$\alpha + \beta + \gamma = 1$
convex combinations	$\alpha, \beta \geq 0$

What do these look like visualized in the plane?

(if we allow *all* possible combinations of each type)



linear combinations: entire plane

affine

$\alpha \vec{x} + \beta \vec{y}$ where $\alpha + \beta = 1$

convex

If vectors can be arbitrarily weird, can we still describe them with numbers?

linearly indep.

basis: $\vec{x}_1, \vec{x}_2, \vec{x}_3, \ldots \vec{x}_n$

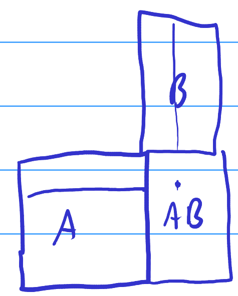
unique coordinates: $\alpha_1, \ldots \alpha_n$ $\quad \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix}$

linear comb: $\alpha_1 \cdot \vec{x}_1 + \alpha_2 \cdot \vec{x}_2 + \cdots + \alpha_n \cdot \vec{x}_n$

Can we use coordinates to describe interesting operations on vectors?

$$\begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \underset{\text{input}}{\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}} = \underset{\text{output}}{\begin{pmatrix} 2\alpha_3 \\ 0 \\ 0 \end{pmatrix}}$$

$$\begin{pmatrix} & & \\ & A & \\ & & \end{pmatrix} \cdot \begin{pmatrix} B \end{pmatrix}$$



$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2\alpha_3 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} c \\ -s \end{pmatrix}$$