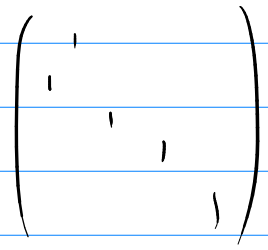
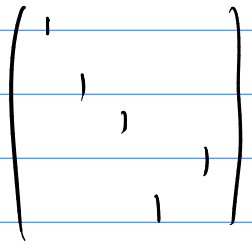
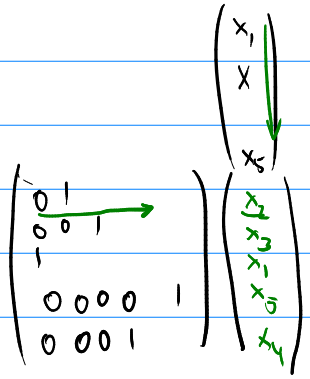
What about products of multiple permutation matrices?

$P_1 =$  $P_2 =$ 

$P_2 P_1$

What happens if you compute PA for a (complicated) permutation matrix?



What happens if you compute AP for a (complicated) permutation matrix?

That has made quite a mess of our LU factorization, right?

$3\times 3 \rightarrow \otimes\ M_2 P_2 M_1 P_1 A = U$ (A: $n\times n$)

$PA = LU$

$L_2 = M_2$

$L_1 = P_2 M_1 P_2^{-1}$

$L_2 L_1 P_2 P_1 = M_2 (P_2 M_1 P_2^{-1}) P_2 P_1$

$= M_2 P_2 M_1 P_1 = \otimes$

$L_2 L_1 P_2 P_1 A = U$

$\Leftrightarrow \underbrace{P_2 P_1}_{P} A = \underbrace{L_1^{-1} L_2^{-1}}_{L} U$

$\Leftrightarrow PA = LU$

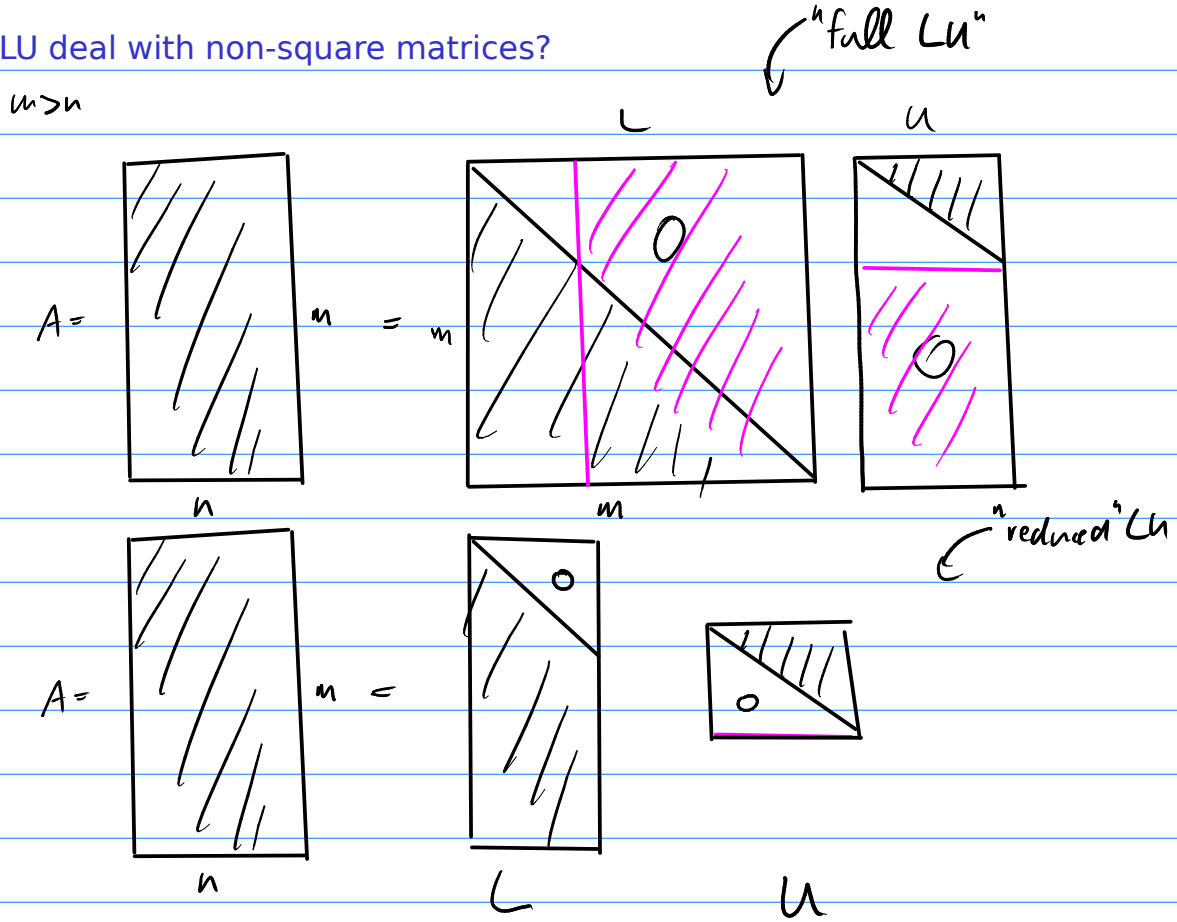
So... how do we sort out this mess?

So... how do we sort out this mess? (cont'd)

Are there any remaining failure scenarios for LU?

What happens if the matrix in LU factorization is not invertible?

Can LU deal with non-square matrices?



Applications of LU

(1) Solve linear equations. How?

(2) Solve a matrix equation. How?

(3) Find the basis of a span. How?

(4) Find the determinant of a matrix. How?

(5) We'd like to find the rank* of a matrix. Is that possible using a computer?

*rank: Number of linearly independent rows/columns

Suppose we take that into account. How would we compute the rank?

(6) Finding the nullspace of a matrix A

(6) Finding the nullspace of a matrix A