What's an orthogonal basis?

What's an orthonormal basis (or "ONB")?

For some given vector
$$\vec{x}$$
, how do I find coefficients with respect to an ONB?

 $\vec{x} + (\vec{z}, \vec{b}), \vec{b}, + (\vec{x}, \vec{b}), \vec{b}, \vec{z}, (\vec{x}, \vec{b}), \vec{b}, + \cdots + (\vec{x}, \vec{b}), \vec{b},$

What if $\vec{x} = \vec{b}$?

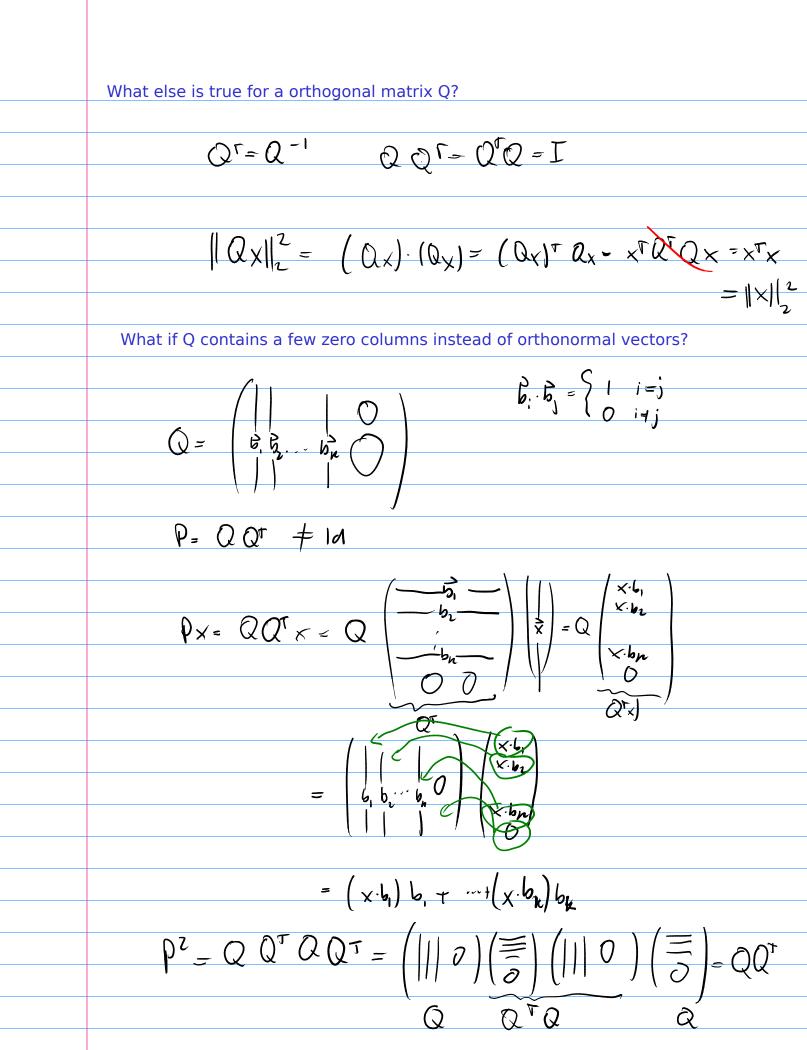
 $\vec{b}, + (\vec{x}, \vec{b}), \vec{b}, + (\vec{x}, \vec{b}), \vec{b}, \pm (\vec{x}, \vec{b}), \vec{b}, \pm \cdots + (\vec{x}, \vec{b}), \vec{b},$

What if $\vec{x} = \vec{b}$?

 $\vec{b}, + (\vec{x}, \vec{b}), \vec{b}, \pm (\vec{x}, \vec{b}), \vec{b}, \pm \cdots \pm (\vec{x}, \vec{b}), \vec{b},$

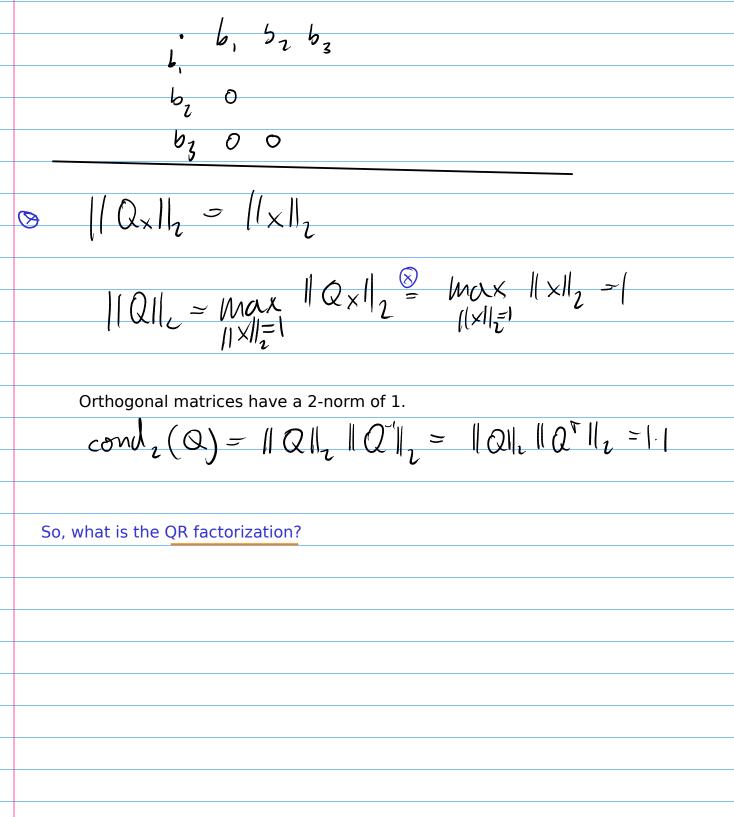
Can we build a matrix that computes those coefficients for us?

Image: Compute the computes the coefficients of t



, לי י 0 <u>b</u>, О С Q[↑]() = C ć





If life were consistent, shouldn't this be called the QU factorization?

What is the cost of QR factorization (for an nxn matrix A)?
Does QR work for non-square matrices?
Is Q still orthogonal in a "thin" QR factorization?

If I have a "thin" QR, can I obtain a "full" QR from it?
 Can QR fail?



(1) Solve square linear systems. How?

(2) Solve tall and skinny linear systems. How?

Is there other notation for least-squares problems?
And how does QR help with least-squares problems?

So how do I solve a least-squares problem with QR?
What about the "normal equations"?

How can I use least-squares problems for fitting a linear model to data?