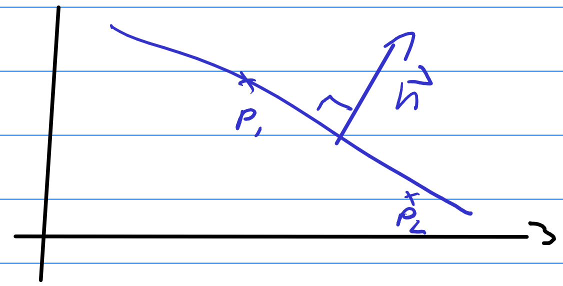
Discuss P1 on the quiz



$$\vec{x}\cdot\vec{n} - r = 0$$

$$\vec{n} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$$

$$\|\vec{n}\|_2 = 1$$

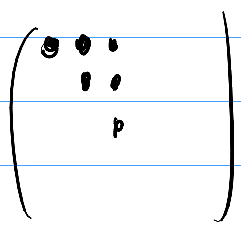
$$\vec{x}\cdot\vec{n} - 2 = 0$$

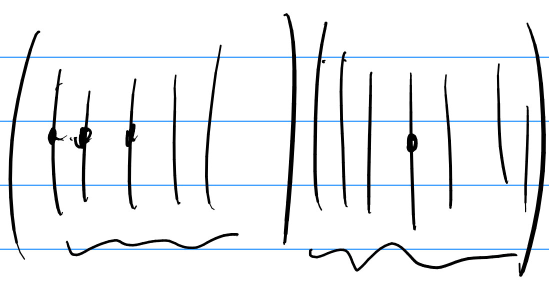
$$\vec{n}\cdot(\vec{p_1} - \vec{p_2}) = 0$$

$$d = (\vec{x}\cdot\vec{n} - 2)$$

$$\underbrace{\begin{pmatrix}1\\0\end{pmatrix}\cdot\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}}$$

$$\frac{1}{\sqrt{2}} - 2$$





Q A

Orthonormal vectors seem very useful. How can we make more than two?

So, what is the QR factorization?

$A = QR$

Q orthogonal

R upper triangular

If life were consistent, shouldn't this be called the QU factorization?

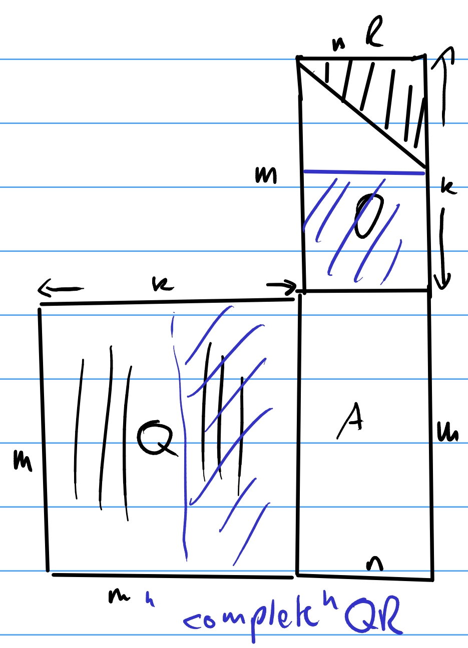
Yes.

What is the cost of QR factorization (for an nxn matrix A)?

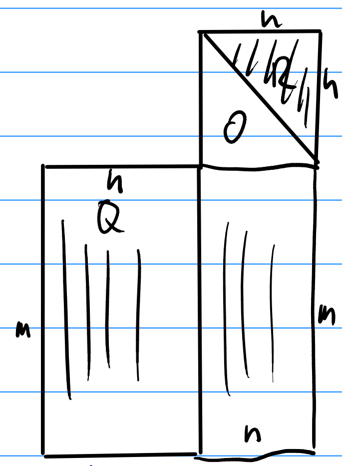
$O(n^3)$

for each column of A: (n)
    q = that column
    for each previous column of Q: (n)
        compute a q . Q-colum (n)
        and subtract off that component
    stick q (normalized) into the next available column of Q

Does QR work for non-square matrices?



$A = QR$



"complete" QR

"reduced" QR

"thin"

Is Q still orthogonal in a "thin" QR factorization?

$QQ^T = I \rightarrow$ no.

Need complete factorization for Q to be *actually* orthogonal

If I have a "thin" QR, can I obtain a "full" QR from it?

Can QR fail?

No – if you catch and ignore division by zero.

6 Applications of QR

(1) Solve square linear systems. How?

$$A = QR$$

$$Ax=b \quad \rightsquigarrow \quad Q\underbrace{Rx}_{y}=b$$

$$\rightsquigarrow \quad Qy=b \quad \big|\, Q^T\cdot$$

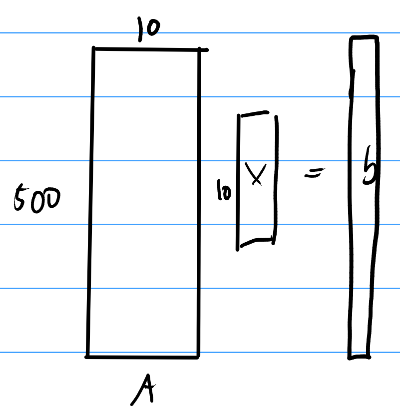
$$\rightsquigarrow \quad \cancel{Q^TQ}\, y=Q^Tb \qquad (O(n^2))$$

$$\rightsquigarrow \quad Rx=y \rightarrow \text{backsubst}$$

$O(n^2)$

(2) Solve tall and skinny linear systems. How?

LU only succeeds if A is invertible (and thereby square)



minimize $\|\underbrace{Ax-b}_{\text{residual } r}\|_2^2 = r_1^2+r_2^2+\cdots r_n^2$

least squares

$Ax \cong b \leftarrow$ solve the LSQ problem:

Is there other notation for least-squares problems?

And how does QR help with least-squares problems?

$A = QR$ $\quad$ A tall, skinny

orth

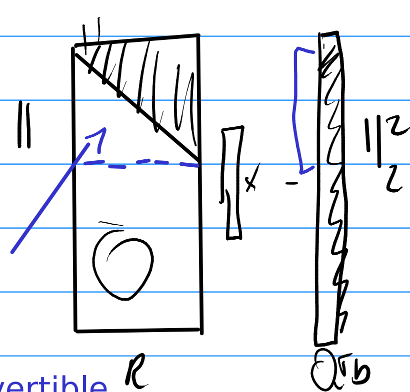
$$\|Q^T x\|_2^2 = \|x\|_2^2$$

$$\|Ax - b\|_2^2 = \|QRx - b\|_2^2$$

$$= \|Q^T(QRx - b)\|_2^2$$

$$= \|Rx - Q^Tb\|_2^2$$

$=$



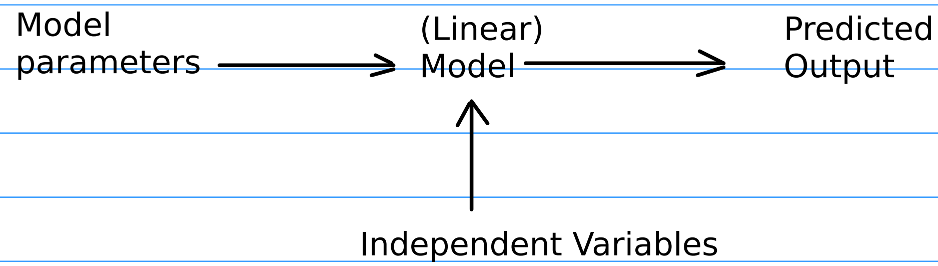
$$= \|R_{upper}x - (Q^Tb)_{upper}\|_2^2 + \overbrace{\|0 - (Q^Tb)_{lower}\|_2^2}^{\text{no } x}$$

$$\|A\underbrace{(x+h)}_{x'}-b\|_2^2 = \|Ax+\underbrace{Ah}_{\partial}-b$$

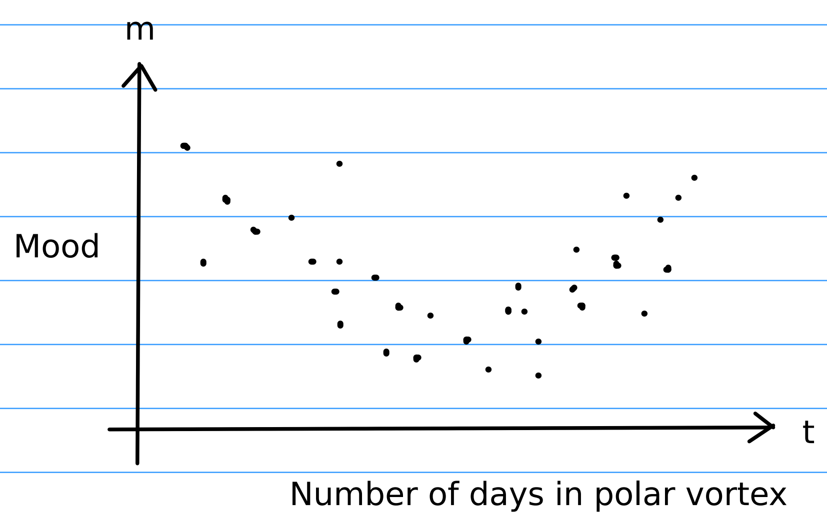
So how do I solve a least-squares problem with QR?

What about the "normal equations"?

How can I use least-squares problems for fitting a linear model to data?



Can you give an example?



How do we find the parameters then?