If I interpolate a function I already know, will the interpolant be exactly that function?

$\otimes$ $$|f(x) - \tilde{f}(x)| \leq C \cdot h^{n+1} \qquad (\text{for all } x)$$

n: highest polynomial degree we've used

In the example: (interpolation with linears)

$$|f(x) - \tilde{f}_h(x)| \leq C \cdot h^2$$

For $\hat{h} = h/2$

$$|f(x) - \tilde{f}_{\hat{h}}(x)| \leq C \cdot \hat{h}^2 = C \cdot \left(\frac{h}{2}\right)^2 = C \cdot h^2 \cdot \frac{1}{4}$$

$\otimes$ only works if two things are true:

- h must be "small enough"
- the function must be "smooth" in the interval

The "best" set of interpolation nodes is called the set of "Chebyshev nodes":

$$x_k = \cos\left(\frac{2k-1}{2n} \cdot \pi\right) \qquad k = 1 \ldots n$$

Monomial: x^i

Polynomial: $x^3 + x^2 + x^1$

Another lesson: Don't try to "overdo it" on the polynomial degree

Polynomial interpolation works best for degrees <=10

So what types of predictions can I make using the interpolation error estimate?

Suppose I have an interpolation error E for some interval length h

for interpolation with a quadratic function.

What would happen if I used h/2 instead of h?

Shorter intervals (smaller h) seem to decrease the error. Why is that?

Interpolation allows several choices. What are good/bad choices?

Have:

- Choice of basis
- Choice of nodes

Let's look at choice of nodes first, then at choice of basis.

What is a "good" set of nodes for polynomial interpolation?

Demo: Choice of interpolation nodes

Observation: Best if nodes cluster towards interval ends

"Best" set of interpolation nodes on [-1,1]: "Chebyshev nodes"

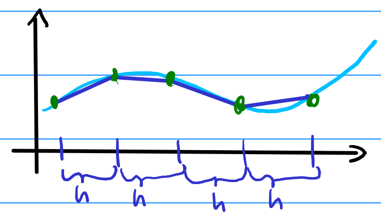
$$x_k = \cos\left(\frac{2k-1}{2n}\pi\right) \qquad k=1\dots n$$

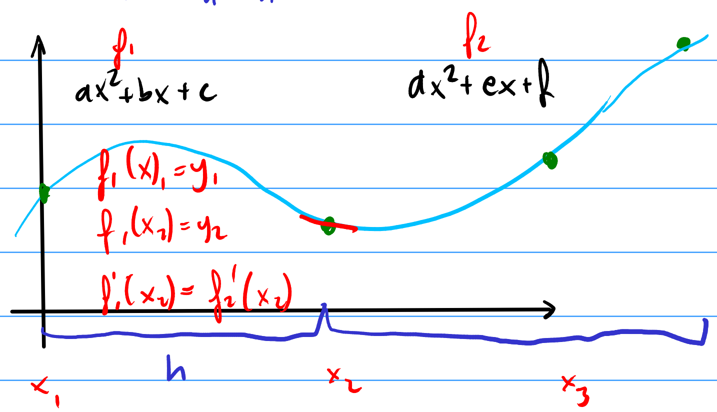
What are some choices of interpolation basis?

What are some choices of interpolation basis? (cont'd)

(2) Piecewise polynomials

For example: Piecewise linear, continuous





$n=2$

$C\cdot h^{n+1}=C\cdot h^3$

$C\cdot\left(\frac{1}{10}\right)^3$

$0.5\cdot\left(\frac{1}{1000}\right)$

What are some choices of interpolation basis? (cont'd)

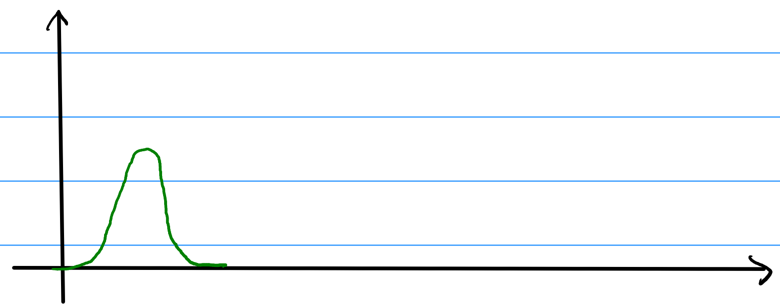
(3) Sines and cosines ("Fourier basis")

What's the basis?

What to use as points?

What are some choices of interpolation basis? (cont'd)

(4) Radial basis functions



So how would I use calculus on an interpolant?

$f(x_i) = \tilde{f}(x_i) \quad i=1,\dots,n$

Have: interpolant $\hat{f}(x) = \alpha_1 \varphi_1(x) + \dots + \alpha_n \varphi_n(x)$

Want: derivative

So how would I use calculus on an interpolant? (cont'd)