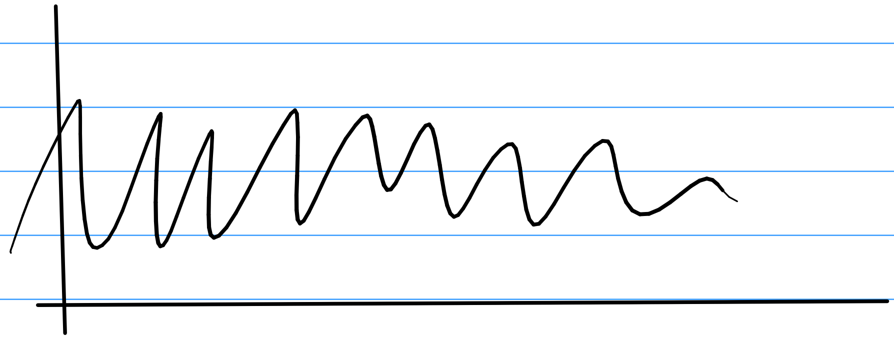
So what types of predictions can I make using the interpolation error estimate?

Interpolation error is small if:

n)

- Function smooth enough

- Vandermonde system well-conditioned

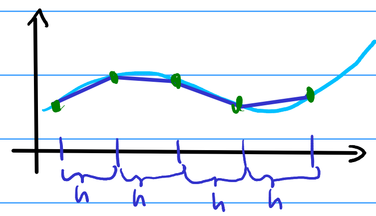


Shorter intervals (smaller h) seem to decrease the error. Why is that?

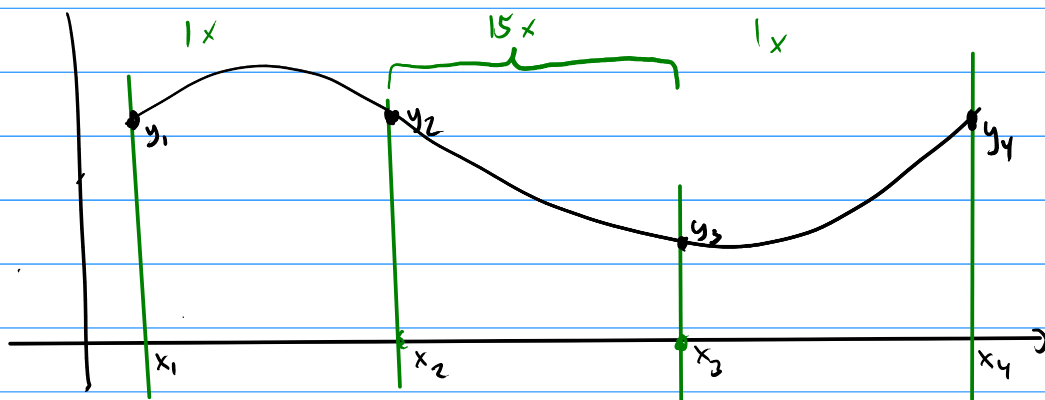
What are some choices of interpolation basis? (cont'd)

(2) Piecewise polynomials

For example: Piecewise linear, continuous



"cubic" spline:



12 unk. → $f_1(x)=a_1x^3+b_1x^2+c_1x+d_1$ | $a_2x^3+b_2x^2+c_2x+d_2$ | $a_3x^3+b_3x^2+c_3x+d_3$

6 equ.:
- $f_1(x_1)=y_1$, $f_1(x_2)=y_2$
- $f_2(x_2)=y_2$, $f_2(x_3)=y_3$
- $f_3(x_3)=y_3$, $f_3(x_4)=y_4$

2 equs.: $f_1'(x_2)=f_2'(x_2)$ $f_2'(x_3)=f_3'(x_3)$

2 equs.: $f_1''(x_2)=f_2''(x_2)$ $f_2''(x_3)=f_3''(x_3)$

c → # chunks   # int. interfaces: $c-1$

unknowns: $4c$

equations: $\underbrace{(c-1)\cdot(\overset{f}{2}+2)}_{4(c-1)}+2$

Still need one condition for each endpoint:

(1) $f_1'(x_1) = f_3'(x_4)$

(2) $f_1''(x_1) = f_3''(x_4) = 0$ ("natural" spline)

$f_1''(x_2) = f_2''(x_2)$

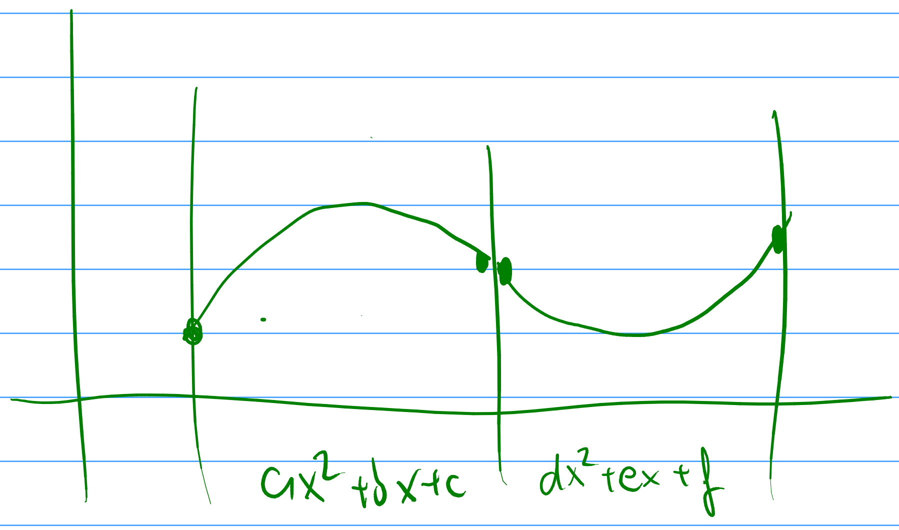
$f_1(x) = a_1x^3 + b_1x^2 + c_1x + d$ $\quad f_2(x) = a_2x^3 + b_2x^2 + c_2x + d_2$

$f_1'(x) = 3a_1x^2 + 2b_1x + c_1$ $\quad f_2'(x) = 3a_2x^2 + 2b_2x + c_2$

$f_1''(x) = 6a_1x_1 + 2b_1$ $\quad f_2''(x) = 6a_2x + 2b_1$

$$\vec{\alpha} = (a_1\ b_1\ c_1\ d_1\ a_2\ b_2\ c_2\ d_2\ \cdots)^T$$

$$A = \begin{pmatrix} 6 \cdot x_2 & 2 & 0 & 0 & -6x_2 & -2 & 0 & 0 \\ & & & & & & & \end{pmatrix} \cdot \vec{\alpha} = \begin{pmatrix} 0 \\ \\ \end{pmatrix}$$



$f_1'(x_0) = 0$

$f_1(x_0) = y_0$

$f_1(x_1) = y_1$

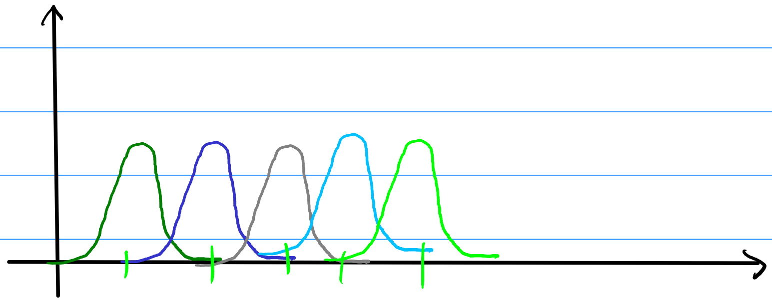
$f_2(x_1) = y_1$

$f_2(x_2) = y_2$

$f_1'(x_1) = f_2'(x_1)$

What are some choices of interpolation basis? (cont'd)

(4) Radial basis functions



Lesson: Radial basis function interpolation *can* work extremely well

BUT: Parameter (Radius) needs to be chosen to trade off accuracy and conditioning

So how would I use calculus on an interpolant?

$f(x_i) = \tilde{f}(x_i) \quad i=1,\dots,n$

Have: interpolant $\hat{f}(x) = \alpha_1 \varphi_1(x) + \dots + \alpha_n \varphi_n(x)$

Want: derivative

So how would I use calculus on an interpolant? (cont'd)

Give a matrix that takes two derivatives.

What is the observed behavior of the error when taking a derivative?

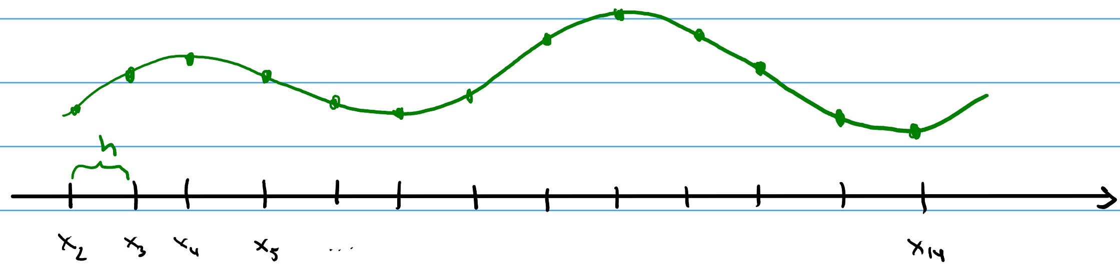
What do the entries of the differentiation matrix mean?

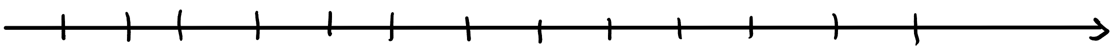
do not care

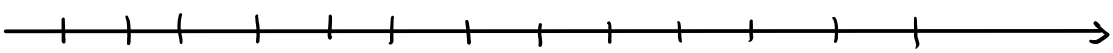
$$D = V'V^{-1} = \begin{pmatrix} \boxed{\quad\quad\quad} \\ \\ \boxed{\quad\quad\quad} \end{pmatrix} \qquad V'V^{-1}\begin{pmatrix} f(x_0) \\ f(x_1) \\ f(x_2) \end{pmatrix} = \begin{pmatrix} \boxed{\quad\quad\quad} \\ \\ \boxed{\quad\quad\quad} \end{pmatrix}$$

$f'(x_1) \approx$

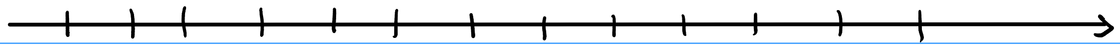
Can we simplify the proccess for equispaced x-coordinates?



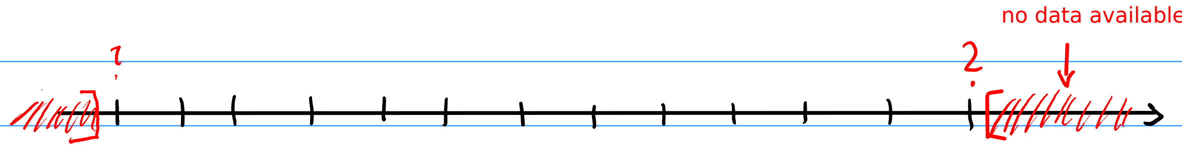




How would we use translation invariance with centered differences?



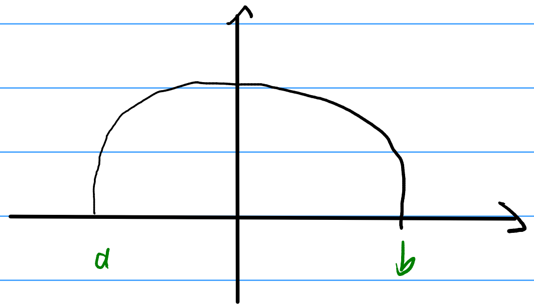
What do we do at the edges?



How accurate are finite difference formulas?

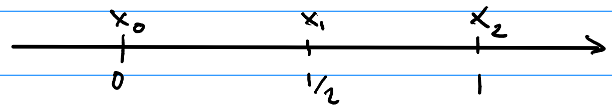
Can anything else go wrong when we numerically compute derivatives?

Can we use a similar process to compute (approximate) integrals?



So, once I know my nodes and my weights, what does quadrature look like?

Can you do a full example?



$f(x_0) = 2 \qquad f(x_1) = 0 \qquad f(x_2) = 3$

1 Find coefficients $\lambda_k =$

2 Compute integrals $\int \quad dx =$

$\int \quad dx =$

$\int \quad dx =$

3 Combine it all together

$\int_0^1 \tilde{f}(x)\,dx =$

What does Simpson's Rule look like on [0, 1/2]?

What does Simpson's Rule look like on [5, 6]?

How accurate is Simpson's rule?