What are some choices of interpolation basis? (cont'd)

(3) Sines and cosines ("Fourier basis")

→ What's the basis? $\cancel{\sin(0x)}$ $\overbrace{\cos(0x)}^{1}$ $\sin(1x)$ $\cos(1x)$

What to use as points? equispaced points between [0, 2pi]

$[0, 2\pi)$ without the point 2pi

$\overbrace{\cos(0x)}^{1}$ $\sin(1x)$ $\cos(1x)$ $\sin(2x)$ $\cos(2x)$ $\sin(3x)$ $\cos(3x)$

So how would I use calculus on an interpolant?

$f(x_i) = \tilde{f}(x_i) \quad i=1 \dots n$

Have: interpolant $\hat{f}(x) = \alpha_1 \varphi_1(x) + \dots + \alpha_n \varphi_n(x)$

Want: derivative $\tilde{f}'(x) = \alpha_1 \varphi_1'(x) + \dots + \alpha_n \varphi_n'(x)$

Have: Function values at nodes x_i $\quad f(x_i)$

$f'(x_i)$ ← have to know calc

$\tilde{f}'(x_i)$

(1) Compute coeffs: $\alpha = V^{-1} \vec{f} \qquad \vec{f} = \begin{pmatrix} f(x_1) \\ \vdots \\ f(x_n) \end{pmatrix}$

(2) Piece together the generalized Vdm of the derivatives of the basis:

$$V' = \begin{pmatrix} \varphi_1'(x_1) & \cdots & \varphi_n'(x_1) \\ & & \\ \varphi_1'(x_n) & & \varphi_n'(x_n) \end{pmatrix} \qquad \vec{\tilde{f}}' = \underbrace{V' V^{-1}}_{\text{Differentiation matrix}} \vec{f}$$

$\varphi_i(x) = x^i \qquad \varphi'(x) = i x^{i-1}$

$x^3 \qquad 3 \cdot x^2$

Interpolation: $|f(x) - \tilde{f}(x)| \le C \cdot h^{n+1}$ (n = highest poly degree)

Differentiation: $|f(x) - \hat{f}(x)| \le C \cdot h^n$

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quad 3coeff cubic 4coeff

$0.1 = E(h) = C \cdot h^n$ $\quad n = 3$

$= E(h/2) = C \cdot \left(\frac{h}{2}\right)^n = C \cdot h^n \cdot \left(\frac{1}{2}\right)^n = \underbrace{E(h)}_{0.1} \cdot \underbrace{\left(\frac{1}{2}\right)^n}_{0.125 = \frac{1}{8}}$