

Can we take norms of matrices, too?

$$\begin{pmatrix} 1 & 2 \\ 7 & 5 \end{pmatrix} \quad \begin{pmatrix} 1 \\ 2 \\ 7 \\ 5 \end{pmatrix}$$

Why does that fall short?

WANT: $\|Ax\| \leq \underbrace{\|A\|}_{\text{matrix norm } \|\cdot\|} \|x\| \leftarrow \text{for any } x$
matrix goes here

Given a vector norm $\|\cdot\|$, define the associated matrix norm in the following way:

$$\|A\| = \max_{\|x\|=1} \|Ax\|$$

$$\|Ax\| = \left\| A \frac{x}{\|x\|} \cdot \|x\| \right\| = \underbrace{\left\| A \frac{x}{\|x\|} \right\|}_{\substack{\| \cdot \| = 1 \\ \leq \|A\|}} \cdot \|x\|$$

$$\leq \|A\| \cdot \|x\|$$

$$\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \rightsquigarrow \begin{pmatrix} 2x \\ y \end{pmatrix}$$