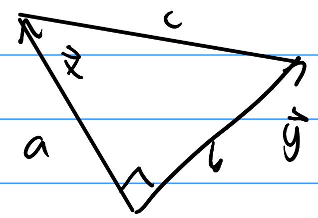
Tell me about orthogonality.

$f(\vec{x}, \vec{y})$ $\quad$ $(\vec{x}, \vec{y})$ $\quad$ "dot" $\vec{x} \cdot \vec{y}$

$\vec{x}, \vec{y}$ orthogonal if $(\vec{x}, \vec{y}) = 0$

$f(\vec{x}, \vec{y}) = 0$ $\quad$ $(\vec{x}, \vec{y}) = 0$ $\quad$ $\vec{x} \cdot \vec{y} = 0$

Pythagorean theorem

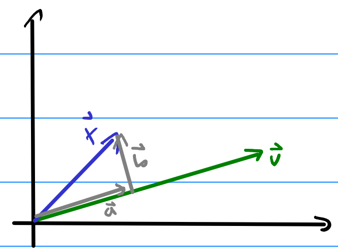


$$a^2 + b^2 = c^2$$

$$\vec{x} \perp \vec{y}$$

$$\|x\|^2 + \|y\|^2 = \|x+y\|^2$$

What if I've got two vectors that are not orthogonal, but I'd like them to be?



Given: $\vec{x}, \vec{v}$

Want: $\vec{a}, \vec{b}$

with $\vec{a} = \alpha \vec{v}$

$\vec{b} \perp \vec{v}$

$\vec{x} = \vec{a} + \vec{b}$

$$\vec{x} = \alpha \vec{v} + \vec{b}$$

$$\vec{b} = \vec{x} - \alpha \vec{v}$$

$$\vec{b} \perp \vec{v} \iff (\vec{b}, \vec{v}) = 0$$

$$\iff (\vec{x} - \alpha \vec{v}, \vec{v}) = 0$$

$$\iff (\vec{x}, \vec{v}) - \alpha (\vec{v}, \vec{v}) = 0$$

$$\iff \alpha = \frac{(\vec{x}, \vec{v})}{(\vec{v}, \vec{v})}$$

$$\vec{b} = \vec{x} - \alpha \vec{v} = x - \frac{(\vec{x}, \vec{v})}{(\vec{v}, \vec{v})} \vec{v} \qquad \vec{a} = \alpha \vec{v} = \frac{(\vec{x}, \vec{v})}{(\vec{v}, \vec{v})} \vec{v}$$

$\vec{x}^{\perp \vec{v}}$ $\qquad$ $\vec{x}^{\parallel \vec{v}}$